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# IS GHETTOISATION A COLLATERAL DAMAGE OF "BALLOT BOX DEMOCRACY" IN THE DEVELOPING WORLD?

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**Abstract**

*In contrast with the evolving racial segregation in the US and Europe over the centuries, the latest phenomenon of urban segregation in the developing world is mostly based on what is commonly known as segregation by income. These urban ghettos are informally built settlements, mostly in the outskirts of large cities, the formation of which is often driven by rural to urban migration as poor migrants cannot afford to pay the premium price for the formal urban housing. Segregation in the developing world has created an unusual dichotomy as, on one side, we see the chaotic world of ghettos while on the other side we see the organized and flourishing advanced sector. The stark dichotomy has prompted reactions from regional and local authorities to physically isolate ghettos from the organized parts of urban centres. We argue that the formation and sustenance urban ghettos, or ghettoisation, in developing nations turn on the pivots of peculiar economic advantages and political opportunism and ours will be a first model to blend these economic and political factors to explain the formation of urban ghettos in equilibrium. We construct a simple game with two ghetto overlords who make relevant economic and political decisions in their respective ghettos. Ghetto overlords are rivals and strategically choose the optimal size of their respective ghettos, while the size of a ghetto determines the local supply of labour and thereby drives the informal economy of a ghetto. The size of a ghetto also endows the ghetto overlord with electoral votes of their ghetto dwellers. With these assumptions and simple functional forms, we characterise the Nash equilibrium of the proposed game. Three important observations are in order: first, the game has multiple equilibria and the equilibria can be Pareto-ranked, which gives rise to the problem of what is commonly known as indeterminacy. Secondly, if a dynamic process is superimposed, then it is possible to examine the stability property of each equilibrium. If the dynamic process is simplistically represented by a first-order difference equation, then conditions under which chaos and cycles would occur are characterised. Finally, we find the bifurcation property of a stable Nash equilibrium can render the ghetto's economic and political outcomes highly fragile and chaotic, which can seriously impinge on the lives of about 500 million ghetto dwellers in developing nations.*

**Key-words**: Competitive ghettoisation; Multiple Nash equilibria; Bifurcation; Fragility and indeterminacy of equilibria

**JEL Classification**: C02; C62; C72; O18; P16

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## 1. Introduction

At the outset it is important to note that we use the word ghetto in a non-pejorative fashion in this work. In the mainstream parlance, the word ghetto implies some kind of segregation between people. However, most of the existing economic studies on ghettos turn on the pivot of one type of segregation, namely, the racial or ethnic segregation. This emphasis on racial segregation derives from the common experience of the US and other developed nations in which residential segregation along racial or ethnic lines regularly take place in their urban landscapes (Borjas, 1995). The black and white cleavage in the US has triggered an extensive literature on the economic and social costs of racial segregation: Case and Katz (1991); Glaeser, Sacerdot and Scheinkman (1996) demonstrated how the peer group pressure determines the work-norms, skill formation and criminal behaviours of youth in segregated ghettos, which thereby perpetuate their economic and social disadvantages. Cutler and Glaeser (1997) in their very important work marshalled evidence to establish the following crucial result: blacks from more racially segregated cities earn less income, have higher school dropout rates and increased risks of being a single parent. Wilson (1997) argued how segregation engenders statistical discrimination that offers incentives for whites to use racial stereotypes against blacks, which further expands racial cleavages in the US.

If one looks at the US history, at the very outset, the formation of ghettos and racial segregation in the US were caused by the migration of newly freed slaves from the South to move northward to establish themselves in mostly white cities of American Midwest and Northeast. The segregation was a natural and historical by-product of exclusion dictated by the racial prejudice of the majority of white populace in cities of American Midwest and Northeast. Since the 1950s we have seen appearance of similar segregation on the European soil as Algerian and Turkish ghettos started dotting the urban landscapes of France and Germany respectively.

In contrast with the evolving racial segregation in the US and Europe over the centuries, the latest phenomenon of segregation in the developing world is mostly based on segregation by income (Gangopadhyay and Nath, 2001). The sprawling ghettos of large Asian cities, the favelas of Brazil, and the gecekondu settlements around Turkey's large cities represent the informal sector that is ubiquitous in cities of developing nations. These urban ghettos are informally built settlements, mostly in the outskirts of large cities, the formation of which is often driven by rural to urban migration as poor migrants cannot afford to pay the premium price for the formal urban housing. At the risk of repetition, segregation by income is a time-honoured phenomenon as highlighted by none-other than Socrates who put forward the classic comment on segregation in Greek Polis in his 'The Republic':

> *"For, indeed any city, however small, is in fact divided into two, one the city of the poor the other of the rich; these are at war with one another; and in either there are many smaller divisions, and you would be altogether beside the mark if you treated them as a single State." (see Jowett, 1999, pp. 137).*

Segregation by income breeds inequality, divisions, ill-feelings and serious social heat that can grossly weaken a city State. Our work will follow these basic notions of Socrates to model some unprecedented phenomena in the context of segregation in the developing world: Our main contributions will be three-fold: first and foremost, segregation by income in developing nations seems to have created very deep cleavages between the rich and the poor, which seems to be as deep and as intense as the traditional racial division of the medieval European communities into Jewish and non-Jewish splits and the split of some of the US cities into blacks and whites following the emancipation and migration of the slaves. Traditionally, in the context of the US and Europe social scientists and observers have relied on the rhetoric of racial hatred to explain involuntary formation of ghettos (see an early work by Bailey, 1959). However, in developing nations such strong racial cleavages are absent, we will therefore develop a special model that helps explain the deep cleavages in cities of developing nations in terms of economic and political factors.

Secondly, the segregation in developing world created a dichotomy: on one side we see the chaotic world of ghettos and the on the other side we see the organized advanced sector. This dichotomy has led to reactions from regional and local governments to physically isolate ghettos from the organized sector: as an example, the government of Rio de Janeiro State proposed to build a 3 meter tall concrete wall around its sprawling favelas in order to separate the chaotic favelas from the picturesque city-centre. This effort is akin to the Israeli action of walling off Palestine to protect Israel from terrorist and suicidal bombings, despite the declaration of the wall as illegal by the International Court of Justice. Our proposed model will explain what can drive regional and local governments to create a barrier between ghettos and the rest of the city in developing nations.

Finally, the formation of ghettos in developing nations is turned on peculiar economic advantages and peculiar political opportunism, or ballot box democracy, and ours will be a first model to blend these economic and political factors to explain the formation of urban ghettos in developing nations.

The proposed model highlights two aspects of a ghetto in cities of developing nations: first, a typical ghetto is an economic powerhouse that specializes in the production of specific goods like leatherwork, leather tanning, illicit drugs and liquor and firework etc. Ghetto dwellers are a cheap source of labour, albeit with a low level of human capital, to carry out these production activities. Hence the size of a ghetto, we argue, impinges on the cost of production of these producers within a ghetto. Secondly, a ghetto acts as a vote-bank for political parties as often ghettos occupy illegal urban lands and carry out illegal trades with the active support of political parties. This second aspect of the democratic system is what we call “ballot box democracy” in which political parties try to influence the electoral outcome by using votes of ghetto dwellers. Typically a ghetto is governed, managed and controlled by the muscle power of different political parties. More often than not, local leaders of these political parties either directly control the ghetto economy, or collect imposts from actual producers known as ‘haptas’ (literally translated weekly tax). A common jargon of ghetto dwellers poignantly summarizes our modeling ideas: “a ghetto dweller is blessed with two hands and a vote”.

In order to model the economics and politics of ghettos, we consider a model with two ghettos, which can easily be extended to a finite number of ghettos. In the existing literature Gangopadhyay (2000) developed a model of a single ghetto to understand the allocation of local public goods within a ghetto. In contrast, in this work, we consider a competitive ghetto formation and construct a simple duopoly with two ghetto overlords (hereafter managers) who make all economic and political decisions in their respective ghettos. What is important for us is that these ghetto managers choose the optimal size of their ghettos. The size of a ghetto determines the local supply of labour and thereby the informal economy of a ghetto. The size of a ghetto also endows the ghetto mangers with electoral votes of their ghetto dwellers.

In the simplest sense, we introduce two modifications in the standard Cournot model of duopoly with linear demand function: first, we make the marginal cost of production of each ghetto managers endogenous and interdependent. Secondly, we introduce the possibility that ghetto managers derive some benefits from their ghetto size, which one may like to call "empire building" motive. We thereby offer an equilibrium size of ghettos in the light of the above extension of the Cournot model.

In order to derive the equilibrium, there are two assumptions that play crucial roles. First, the average cost of production is independent of the output level, but is dependent on the labour supply in a ghetto. This dependence is expressed as a U-shaped relationship, reflecting the notion that as the labour stock reaches a critical level, control, governance and supervision cost of ghetto managers becomes very high and thus adversely affects the per unit cost of production. Second, ghetto managers have an objective function that is a linear combination of economic profits and the size of their ghettos. Under these assumptions, it is possible to have multiple equilibria for the overall game. If a dynamic process is superimposed, then it is possible to examine the stability property of each equilibrium. If the dynamic process is simplistically represented by a first-order difference equation, then conditions under which chaos and cycles would occur can be characterised. The plan of the paper is as follows: in Section 2 we offer the model and conclude in Section 3.

## 2.1 Simple Model of Ghetto Economy

By construction of the problem, we have two ghettos in a city. Both these ghettos produce an identical good and there is a local market which is characterised by a simple linear inverse demand function, P=1-Q, where P is the price and Q is the quantity produced of the good in question. The industry is characterised by a duopolistic rivalry between these two ghettos. The average/marginal cost of production in ghetto j is labelled as $c_j$. We posit that the cost of production is a function of the ghetto size, $l_j$, which is assumed to be the size of the labour force. We express this as

$$c_j = f(l_j) \quad \text{... (1a)}$$

A ghetto is postulated to be an alliance formation between local politicians and ghetto decision makers, who control two groups of production units $M_1$ and $M_2$. We still don't know the size of each group (size of labour force), which derives from the optimal choices made by ghetto managers. By construction, we call $l_i$ the size of ghetto $M_i$, which is by construction is the size of local labour input controlled by ghetto i. The choice variable of a ghetto decision-maker is $l_i$. The equilibrium size of the ghetto is defined as the combination of mutual best responses of a duopoly $M_1$ and $M_2$ in terms of their $l_1$ and $l_2$. Dubbing these two (unknown size and memberships) ghetto decision-makers as two rivals in the local product market, with their marginal costs $c_1$ and $c_2$, the Nash-Cournot output ($Q_i^*$), price ($P^*$) respectively are $Q_i^*=(1-2c_i+c_j)$, $P^*=(1+c_1+c_2)/3$, and profits ($\pi_i^*$) are given as

$$\pi_i^*=(1-2c_i+c_j)^2/9 \quad \text{... (1b)}$$

As a ghetto manager i chooses $l_i$, the average cost of production $c_i$ changes. As the cost of each group changes, it alters their relative position in the local product market. The incentives to alter $l_i$ partly depend on this change in the relative position of ghetto i that is driven by a change in cost function. Each ghetto manager strategically chooses $l_i$ to acquire the best possible market share given the anticipated $l_j$ of rival j. The increase in $l_i$ also offers political gains to ghetto manager i.

## 2.2. Competitive Ghettoisation in a “Ballot Box Democracy”: Fragility and Indeterminacy of Equilibrium Ghetto Size

In this section we will specify two important elements to model the proposed Nash equilibrium of the proposed game of the previous section. The first is in the cost function that takes into account a supervision constraint that a ghetto manager faces as the size of the ghetto increases. Secondly, we introduce a utility function of the ghetto manager that will specify the managerial compensation function, which is in turn influenced by the ballot box democracy. We retain the usual assumption that ghetto managers are compensated on the basis of their performance measured in terms of own-ghetto profits. We introduce the idea that ghetto managers also derive benefits from the size of their ghetto mainly from the votes of their own ghetto dwellers. In other words, the ballot box democracy represents the political power of ghetto managers in influencing, and even controlling, the voting behaviour of their ghetto residents.

Note that the analysis in the previous section has been based on an unspecific post-alliance cost function as given in equation (1). In order to highlight the finite limitation on the supervisory capacity of ghetto mangers we introduce a specific cost function of a ghetto that is U-shaped:

**Assumption 1:** As the size $l_i$ of a ghetto $M_i$ increases the average cost $c_i$ declines till a particular level of ghetto size $l_i$ and then starts rising. That is,

$$c_i=c_0-c_{11}l_i+c_{12}l_i^2 \text{ and } \quad c_{11}>0, c_{12}>0 \qquad \text{... (2)}$$

Hence $(dc_i/dl_i)<0$ for $l_i<[c_{11}/(2c_{12})]$ and $(dc_i/dl_i)>0$ for $l_i>[c_{11}/(2c_{12})]$. We assume that the structure of the average cost is the same for each ghetto.

The analytical results in this paper depend on the specification of the average (or marginal) long-run cost functions being quadratic in nature. Note that the ghetto managers simultaneously choose quantities ($Q_i$) and ghetto sizes ($l_i$), thus a point on the long-run average cost function. It is important to note that the qualitative results only require non-linear cost functions, whilst the quadratic cost functions allow us to derive exact results of multiplicity of equilibria and their stability properties. Yet the specification of quadratic cost function is more important than just for the sake of analytical tractability and convenience. One may like to argue that production activities in ghettos can give rise to economic of scales and scope, vertical integration, learning-by-doing and non-constant bargaining power of ghetto managers over input-owners. In other fields, these elements have led the profession to construct and fit quadratic cost functions for industries dominated by large producers (see Roller, 1990; Halvorsen, 1991; Pulley and Braunstein, 1992 for the relevance of this specification). Early work by Christensen and Greene (1976) established how quadratic average cost function is driven by varying real costs of energy inputs in electric power industry, and not by economies of scale. In the context of financial industries dominated by big firms, early work of Wilson (1981) estimated the quadratic average cost function in the savings and loans industry in the US, on the basis of which important restructuring was initiated. Quadratic average (marginal) cost functions have been successfully applied to characterise cost conditions in industries dominated by big players as in chemical industry (Lieberman, 1984), high-tech industries (Irwin and Klenow, 1994), aircraft manufacturing (Bittlingmayer, 1990), automobile and energy-producing industries. Minimum efficient scales have been derived for oil, chemical and steel industries (Siberston, 1972), petroleum refining (Stigler, 1968) and beer plants (Elzinga, 1986). Church and Gandal (1992) argued how network externalities in production, mainly in supporting services, can play an important role that can in turn explain the quadratic average cost function. The most difficult problem with the quadratic specification was that the specification violates an important property of cost functions being homogeneous of degree one in input prices (see Halvorsen, 1991), which has been addressed in the "properness" definition in Roller (1990).

**Assumption 2:** The compensation, $T_i$, of ghetto manager i is postulated to be:

$$T_i=ml_i+\pi_i^* \quad \text{... (3a)}$$

The first component is the managerial return from the size of the ghetto, which increases with $l_i$ at a constant rate m. The second component is the usual 'own-firm' profits.

The profit functions are arrived at after substituting the cost function (2) into the profit function (1b):

$$\pi_1^*=[(1-c_0+(2l_1-l_2)c_{11}-(2l_1^2-l_2^2)c_{12})]^2/9 \quad \text{... (3b)}$$

$$\pi_2^*=[(1-c_0+(2l_2-l_1)c_{11}-(2l_2^2-l_1^2)c_{12})]^2/9 \quad \text{... (3c)}$$

In what follows we only consider a symmetric equilibrium, for the sake of simplification, and assume $l_1=l_2=L$

**Statement 1:** The optimal size of ghetto i ($L=l_i$) in response to the rival's optimal size the ghetto ($L=l_j$) is given by:

$$9m/[4(2c_{12}L-c_{11})]=1-c_0+c_{11}L-c_{12}L^2 \quad \text{... (4)}$$

(4) can be re-written as:

$$L= \beta_0/\beta_1-\beta_2/\beta_1 L^2 + \beta_3/\beta_1 L^3 \quad \text{... (5a)}$$

$$\beta_0=(1-c_0)c_{11}+(9/4)m,\ \beta_1=2(1-c_0)c_{12}-c_{11}^2,\ \beta_2=4c_{11}c_{12},\ \beta_3=4c_{12}^2 \quad \text{... (5b)}$$

Thus, one can express (5b) as

$$L= H(L)=\beta_0/\beta_1-\beta_2/\beta_1 L^2 + \beta_3/\beta_1 L^3 \quad \text{... (5b}'\text{)}$$

Simple substitutions yield the result. Equation (5a) is the (implicit) reaction function of each of these two ghetto managers.

From the construction of the problem, L is the size of each ghetto. Now we consider the equilibrium ghetto size from the reaction function (5b′). We apply the simple notion of the Nash equilibrium: the equilibrium Ls represent the mutual best responses of ghetto managers. By construction, the Nash equilibrium is given by the fixed point of equation (5b′). In order to characterize the equilibrium ghetto size we need the following details.

**Observation 1**: Define L** as

$$L^{**}= (2\beta_3)/(3\beta_2)=(2c_{11})/(3c_{12}) \quad \text{... (6a)}$$

$$\text{For } 0<L<L^{**},\ H'(L)<0 \quad \text{... (6b)}$$

$$\text{For } L>L^{**},\ H'(L)>0 \quad \text{... (6c)}$$

Simple differentiation of H(L) with respect to L yields the above.

**Observation 2:** The fixed point is the point of intersection between H(L) function and the 45 degree line passing through the origin. From Diagram 1 we note either there exist two equilibrium ghetto size $E_1$ and $E_2$, or there does not exist any equilibrium. This is so since $(\beta_0/\beta_1)>0$, $(\beta_2/\beta_1)>0$ and $(\beta_3/\beta_1)>0$ for being economically meaningful (profits being positive). As a result, the H(L) function is inverse bell-shaped. There is no equilibrium if

$$H(L^{**})>(2c_{11})/(3c_{12})=L^{**} \quad \text{... (6d)}$$

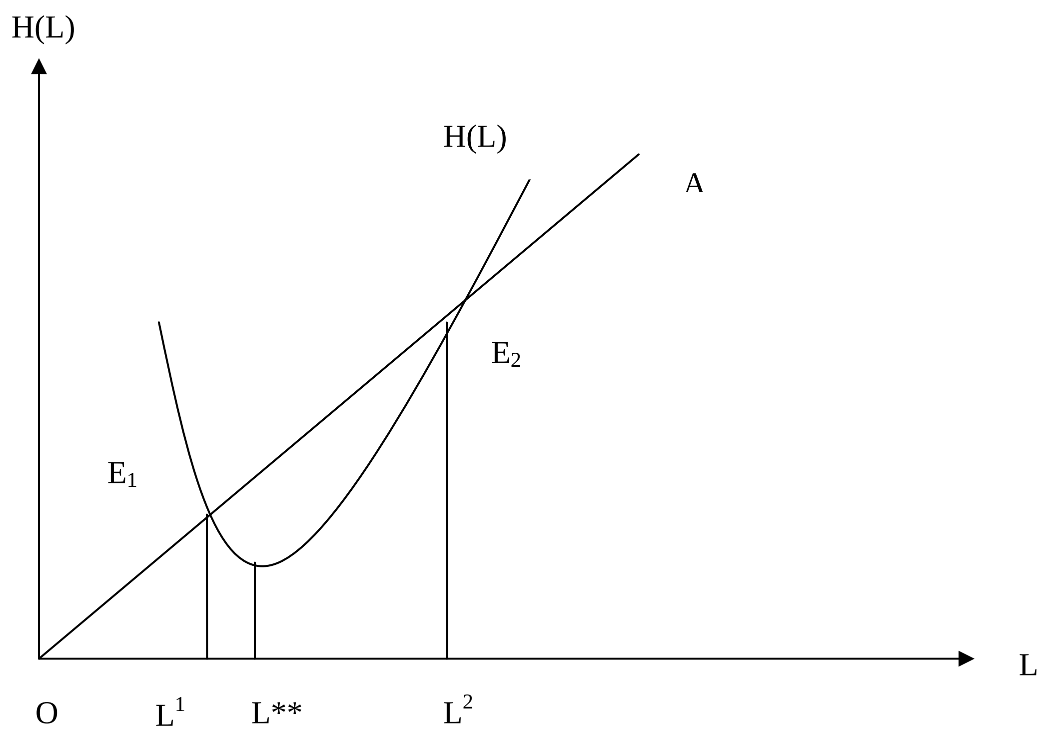
On the other hand there exist two equilibria if:

$$H(L^{**})<(2c_{11})/(3c_{12})=L^{**} \quad \text{... (6d')}$$

Where

$$H(L^{**})=[\ \beta_0/\beta_1-\beta_2/\beta_1\ [(4c_{12}^2)/9c_{11}^2)] + \beta_3/\beta_1\ [(8c_{12}^3)/(27c_{11}^3)] \quad \text{... (6d'')}$$

Proof: Note that $H'(L)=[3L\beta_3-2\beta_2]L/\beta_1$ and $H'(L^{**})=0$ and $H'(0)=0$ and $H''(L^{**})=[\ \beta_2/\beta_1]>0$. Hence H(L) reaches its local minimum at L** and local maximum at L=0. As a result, two fixed points are positive and the third fixed point is negative. The above can be confirmed from Diagram 1 since H(L) is an inverse bell-shaped function. QED.




Note: OA is the 45 degree line and the fixed points are $E_1$ and $E_2$ provided the sufficient condition is satisfied. Note that $E_1$ may be stable, or unstable, and $E_2$ is always unstable. Note that $L^1$ and $L^2$ are the equilibrium sizes of ghetto corresponding to $E_1$ and $E_2$.

**Diagram 1. Multiplicity of Equilibrium Ghetto Sizes**

**Statement 2:** There are two Nash equilibria, one is shown as a high-ghettoized equilibrium (E2 and L2) and the other is a low-ghettoized equilibrium (E1, L1). Equilibrium E2 is an unstable and Equilibrium E1 is stable if $|H'(L1)|<1$. However, E1 is unstable if $|H'(L1)|>1$. As a result, if $|H'(L1)|>1$ and if the initial value of L lies in the interval [0, L2], the dynamics of ghetto formation will take the system to L=0. One may call L=0 the ghetto-free outcome. If the initial L is greater than L2, then the dynamics of the system takes the system to L=L+, where L+ represents the maximum feasible size of a ghetto.

Thus, the instability property of the high-ghettoized equilibrium retains the possibility of reaching the maximum feasible ghetto size, if history or expectations dictate so (Krugman, 1991a). In order to get a clear picture of a ghetto reaching the maximum feasible size, a possible good example is Dharavi – the largest ghetto of Asia: over a million and a half people are crammed into rows of makeshift shanties, cobbled together with nothing more than asbestos sheets, plastics, bamboo sticks, discarded canvas bags, wooden planks and old car tires. The density of population is about 35 people per square feet. The land is unhealthy and marshy with little toilet facilities and next-to-nothing water supply.

If both equilibria are unstable, the system becomes highly fragile and the initial condition, or history, and expectations can destabilise the system. The instability of equilibria can engender complex dynamics and trigger chaos. Note that the complex dynamics can characterise the size of each ghetto, its economic condition, the size of its labour force. This element of unpredictability of a ghetto economy, along with its myriads of social and economic problems, can prompt local decision makers to take extreme steps to cut out a ghetto from the traditional urban economy: as an example one may like to re-consider attempt of the government of Rio de Janeiro State to build a 3 meter tall concrete wall to separate the chaotic favelas from the central business district of Rio de Janeiro city.

**Observation 3:** Even when the low-ghettoized equilibrium E1 is a stable equilibrium, parametric changes can cause a loss of stability for E1 and the dynamics will take the system either to L=0 or to L=L+. We consider such bifurcation properties in subsection 2.3.

## 2.3 Stability Property of the Equilibrium ($E_1$)

In order to understand the stability property of the equilibrium it is useful to re-construct the fixed point problem (5b″) as

$$L * L = [\, L\Delta_1 - \Delta_0)]/[((L - \Delta_2)] \quad \text{... (6e)}$$

$$\Delta_1 = \beta_1/(3\beta_3),\ \Delta_0 = \beta_0/(3\beta_3),\ \Delta_2 = (2\beta_2)/(3\beta_3) \quad \text{... (6e')}$$

Note that * denotes multiplication and the LHS of (6e) is an increasing function of L as drawn in Diagram 2. The right hand side is a decreasing function if

$$\Delta_0 < \Delta_2\, \Delta_1 \quad \text{... (6e'')}$$

The intersection between these two functions ensures the existence of an equilibrium. This equilibrium exists as long as $(\Delta_0/\Delta_2)>0$ and inequality (6e″) holds. In Diagram 2 we label this equilibrium size of ghetto as $L^1$ in terms of these arguments. The important issue for us is if this equilibrium is stable.

**Corollary 1:** The low-ghettoized equilibrium $L^1$ is stable if

$$L^1(k_1-\Delta_0)^2 > |\, \Delta_0 - \Delta_2\Delta_1 | \quad \text{... (7a)}$$

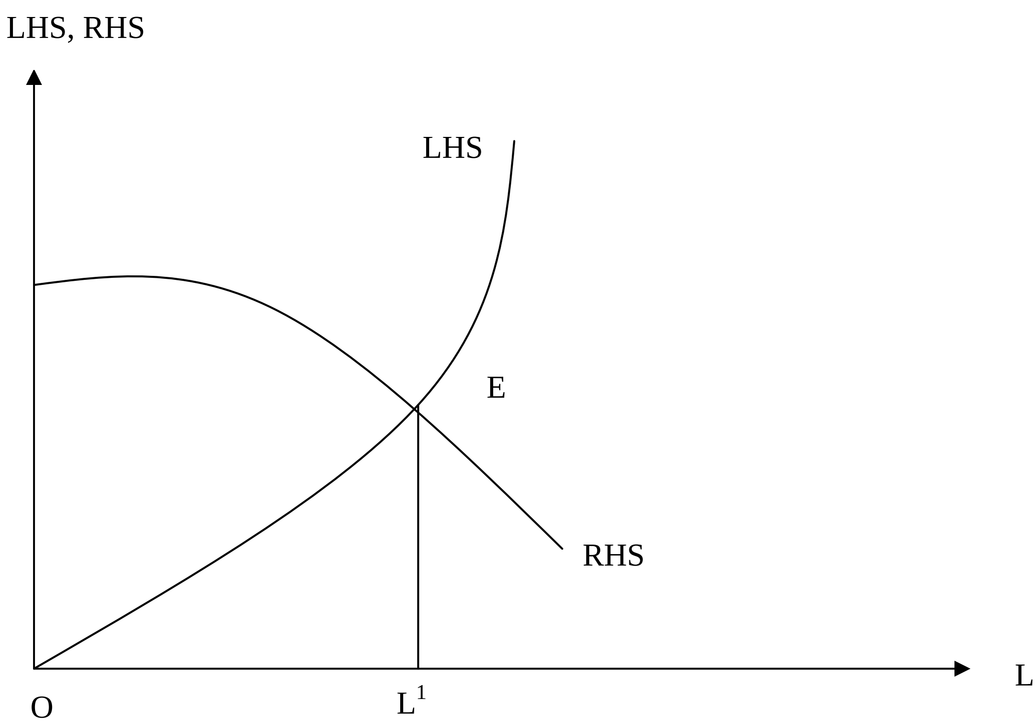
where | | labels the absolute value. Inequality (7a) can be reduced to

$\Delta_0 < 2L^1(L^1-\Delta_0)^2 + \Delta_2\Delta_1$ ... (7a′)

The sufficient condition for (7a′) to hold is:

$m < [4(1-c_0)/(9) - c_{11}^2/c_{12}]c_{11}$ ... (7a″)

Proof: For stability we need at $L_1$, the slope of the LHS of equation (6e) is greater than the absolute value of the slope of the RHS and inequality (7a) ensures this. Expansion of (7a) by appropriate substitutions yield (7a′) and (7a″). QED.




Note: RHS is the right hand side of equation (6e) and LHS is the left hand side of equation (6e). Condition (6e') and (6e") ensure that RHS is downward sloping and hence the equilibrium merger $L^1$ exists. This equilibrium also requires $\Delta_0/\Delta_2>0$. If $\Delta_0/\Delta_2<0$there does not exist any equilibrium ghetto size. The stability of the equilibrium E depends on the values of relevant parameters.

**Diagram 2. Sufficient Condition for a Unique Equilibrium and Bifurcation**

## 2.4 Bifurcation of the Low-Ghettoized Equilibrium

From (7a′) we know the existence of a critical value of $\Delta_0$, $\Delta^*$, where

$\Delta^* = 2L^1(L^1-\Delta_0)^2 + \Delta_2\Delta_1$ ... (7a‴)

Substitution converts (7a′) as

$(1-c_0)c_{11} + 9m < 6\beta_3 L^1 (L^1-\Delta_2)^2 + 3\beta_3\Delta_1\Delta_2$ ... (7b)

As long as $\Delta_0 < \Delta^*$, or the left hand side of (7b) is less than the right hand side of (7b), the low-ghettoized equilibrium $L^1$ is stable. Due to parametric changes if the left hand side of (7b) exceeds the right hand side of (7b), the low-ghettoized equilibrium $L^1$ loses its stability.

**Statement 3:** Given the parameters of the above system, there is a critical value of m, m*, given by

$$m^* = [-(1-c_0)c_{11}]/(9)+[6\beta_3 L^1(L^1-\Delta_2)^2+3\beta_3\Delta_1\Delta_2]/(9) \quad \text{... (7c)}$$

$$= [(1-c_0)/(3)c_{11}^2/(2c_{12}^2)-1/(3)]-[c_{11}^3/(12c_{12}^4)]+[L^1(4L^1c_{12}^2-c_{11})^2/(24c_{12}^5] \quad \text{... (7c')}$$

For m>m*, ceteris paribus, the low-ghettoized equilibrium $L^1$ loses its stability. In a similar fashion we can derive the critical value $c_0$, c*, from (7b) and explain the bifurcation, or loss of stability of low-ghettoized equilibrium $L^1$ if $c_0$>c*.

Proof: The proof is arrived at by examining the stability of the fixed point.[2] QED.

We thus find that the stability of the equilibrium ghetto size depends on the values of the parameters of the system. For example, if the ghetto manager i attaches too much weight to the size of the ghetto (neglecting profits), such that m>m*, the low-ghettoized equilibrium loses its stability. The low-ghettoized equilibrium is also beset with bifurcation problems if the average cost is above a critical value c*, that is, if the cost is too high.[3]

## 4. Ghettoisation: Concluding Comments

Our model establishes some interesting properties of the equilibrium ghettoisation: First, there exist multiple Nash equilibria: one is a high-ghettoized equilibrium (E2 and L2) whilst the other represents a low-ghettoized equilibrium (E1, L1). We showed that the high-ghettoized Equilibrium E2 is always unstable and the low-ghettoized Equilibrium E1 is stable only under a condition. This equilibrium E1 becomes unstable if this condition is violated. If both these Nash equilibria are unstable and if the initial value of ghetto size (L) lies in the interval [0, L2], the dynamics of ghetto formation will take the system to a ghetto-free outcome that is labelled as L=0. If the initial size of ghetto L is greater than a critical value L2, then the dynamics of the system takes the system to a mega ghetto, with size L+, like Dharavi of Mumbai with little civic facilities for its dwellers. We find the bifurcation properties of a stable Nash equilibrium are responsible for making the ghetto's economic and political systems highly fragile and even chaotic.

It is well-known that a chaotic economic system is a mathematical representation of a crisis-ridden and unstable economic system. As a result, our model is constructed to locate a region of stability against the backdrop of a region of instability, which can explain the fragility of a

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2 Note that for m=0, the optimal choice of L by each manager is the efficient scale of operation at which the long-run average cost is minimised. This is obvious by setting m=0 in equation (5a) and from the second order condition of profit-maximisation.

3 What is interesting is that the multiple equilibria and the low-ghetto-size equilibrium can display complex behaviour even when the demand function is linear and the cost function is quadratic in scale. It is anticipated that more general functions can support our finding as long as: 1) the reaction functions of ghetto managers in terms of L are hill-shaped; 2) The Baumol-Benhabib condition holds as a sufficient condition. Our future research will establish sufficient conditions for generating chaotic behaviour when demand functions are non-linear and the cost of production have more complex functional forms in terms of scale/capacity of operation.

ghetto economy.[4] Obviously, the model is a baseline one and there is a strong need to do more research to relate economic crises to unstable equilibria (see Saperstein, 1994). Despite the fact that this is a limited model characterised by linear demand and quadratic cost functions, Footnote 2 explains how the results are more likely to occur with more general functional forms.[5] We also establish the possibility that the competitive ghetto formation, or ghettoisation, as modelled can display both fragility as well as stability in circumstances driven by the values of relevant parameters (see Saperstein, 1984).

There are several questions that will need future attention: the first one is about the appalling quality of life that ghetto dwellers usually live in urban ghettos of developing nations. In Brazil Gecekondu is literally translated as “night-perched” that implies a shabby dwelling carelessly built on public land in one night. The largest slum of Asia, Dharavi in Mumbai, India, is home to 1.5m people on .67 square miles of land. Dharavi’s gross population density is about 35 square feet of land per person, a perfect example of a shanty town in one of the biggest metropolis of our world. It is important for the future research to unravel the sources of this abysmal poverty: there is a reason to believe that the poverty and marginalization of the ghetto dwellers is an effective means to control people for economic and political benefits for the reach and the powerful. Secondly, is there any effective way to improve the economic and social conditions of ghetto dwellers? If the abject poverty in ghettos of the developing world is a means to create a pool of cheap labour and a manipulable vote bank for a “ballot box” democracy, there is little incentive politicians have to improve economic conditions in ghettos. Since a typical ghetto in a developing country is often believed to be a home-ground for criminal activities and even a fountainhead of global terrorism, there must be a global effort to ameliorate the economic plight of 500m ghetto dwellers in developing nations.

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4 Our argument is similar to the “edge of chaos theory”. Stacey (1991) adopted the concept of “edge of chaos theory” for social sciences to argue that sustainability of a social system is achieved by the relevant decision-makers striving to remain at the confluence of the predictable “order” and an unpredictable level of chaos. Chambers (1997) employed this concept of edge of chaos to explain the sustainability of rural communities in developing nations.

5 Ours is a first attempt to provide a simplistic model that can be strained to explain the complexity associated with the proposed Nash equilibrium of a modified Cournot model in which firms endogenously choose their cost of production as a strategic variable.

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