Journal of Quantitative Economics, Vol. 12, No.2, July 2014

TOWARDS THE IDENTIFICATION OF CROSS-COMMODITY RELATIONSHIPS IN METALS MARKETS

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Abstract

This article deals with a cointegration analysis commodity by commodity in metals markets (e.g. Gold, Silver, Platinum, Aluminum, Copper, Nickel, Zinc, Lead) during 1993-2011 using daily data. The main focus lies in the determination of the long term relationship – if any – within precious and industrial metals. We carry out cointegration analyses with/without structural breaks for specific groups of metals commodities. To sum up the main findings, for each of the pair (or more) of commodities investigated, it has been possible to detect at least one cointegration relationship. Therefore, we could broadly conclude that there are more cross-commodity linkages in metals markets than usually thought by market practitioners.

Keywords: Metals Futures; Cointegration; Cross-Commodity Relationship; Structural Break

JEL Classification: L61; Q00; C32; C58

1. Introduction

This article aims at investigating long-term relationships that are common to various segments of metals markets (e.g. precious and industrial metals), as well as the ability of the econometrican to build models incorporating such features.

Cross-commodity relationships imply that two or several commodities share an equilibrium that links prices in the long run. The existence of inter-commodity equilibrium usually indicates long-term co-movement among commodity prices. Temporary deviations from this equilibrium (because of demand and supply imbalances caused by macroeconomic factors and inventory shocks, etc.) will be corrected over the long-run.

These long-term connections – or inter-commodity equilibrium as denoted by Casassus et al. (2009) – include for instanceproduction relationships, where upstream and downstream commodities are tied in the production process. One commodity can be produced from another commodity when the former is the output of a production process that uses the other commodity as an input factor. Alternatively, we may consider substitute/complementary relationships where

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two commodities serve as substitutes or complements. A substitute relationship exists when two traded commodities are substitutes in consumption.A complementary relationship exists when two commodities share a balanced supply, or are complementary in consumption and/or production. Consider for instance the case of industrial metals, which are seldom used in pure form. Most applications can be found in the form of alloys: bronze (tin and copper), soft solder (tin and lead), or pewter (tin and lead). Nickel stocks are used in stainless steel, an alloy of steel. Zinc is often applied as zinc coatings, jointly used with aluminum. in consumption and/or production.

Since the financial crisis of 2008, the topic of cross-marketlinkages(including commodities) has attracted a considerable attention. Some studies analyse the extent of cross-market linkages over different asset classes: stocks-bonds-oil-gold and real estate markets (Chan et al., (2011), metals and energy (Chng, (2009), gold and stocks (Hood and Malik, (2013), energy-food and gold (Mensi et al., (2013). Whereas the econometric methodologies sometimes differ from one study to another, the global conclusion gears towards the frequent identification of cross-market links in recent empirical studies.

One of the most popular economic models that analyzes long term relationships among variables is the cointegration model or Error Correction Models (ECM). This model allows the researcher to analyze the balance of long period relationships. Cointegration is interpreted as a long term relationship because cointegrated variables are tied to each other to keep certain linear combinations stationary, and hence they tend to move together. Nakajima and Ohashi (2011) interpret cointegration among commodity prices to reflect underlying production technologies and firms' activities. The idea behind cointegration is that there is a meaningful reason for commodity prices to move together in the long run, despite non-stationary departures from this equilibrium relationship in the short run. A major advantage of cointegration analysis is that it allows for the possibility that commodity prices in two different markets may respond differently to new market information in the short run, but would return to a long-run equilihrium if both are efficient. There are several reasons to explain why one might expect asymmetric responses from different markets in tbe short run. One is that the markets may have different access times to the information being delivered. Another is that the information may be interpreted differently initially. However, because the commodities trade on common trends, arbitrage opportunities between the markets would eventually result in a multi-market consensus concerning the value of new information.

Many studies have documented empirically cointegrating relationships in metals markets. Aruga and Managi (2011) test the international linkages in the platinum-group metal futures markets. Cortazar et al. (2008) have studied the statistical relationship among commodities in a multi-commodity framework using futures prices. Akram (2009) reveals that different pairs of the commodity prices may be cointegrated. However, none of these papers have provided a thorough analysis of cointegrating relationships within specific groups of metal commodities (e.g. precious and industrial metals).

Commodity markets do not only display wide price fluctuations reflecting demand and supply disequilibria, they also support trading in futures and options whose prices fluctuate as much as stock prices. Market specific factors such as climate and geography, or global demand shocks due to government policies can also contribute to explain why commodity prices are expected to fluctuate together, at least in the long term (Stevens (1991)). In addition, Reinhart and Wickham (1994) enumerate how world commodity prices respond to effects related mainly to policy issues such as stabilization funds, international commodity agreements, external compensatory finance, etc.

There are three related issues to the idea of cointegration in commodity markets that have been deliberately left out of the analysis in this article. First, Pindyck and Rotemberg (1990) offer numerous statistical tests confirming that the prices of several commodities (such as wheat, cotton, copper, gold, crude oil, lumber and cocoa) have a persistent tendency to move together. Co-movement between futures prices exists when two or more prices move together in the long run. Its existence implies that the discovery of one price will provide valuable information about others.As one potential explanation for these excess co-movements, they point out that commodity price movements are partially the result of herd behaviour, i.e. traders who are alternatively bullish or bearish across all commodity markets without justification from economic fundamentals. More recent results on this topic can be found in Chunrong et al. (2006)..

Second, market efficiency implies that the current futures price of a commodity futures contract expiring in $t+1$ should equal the commodity spot price expected to prevail in $t+1$, i.e. that the current futures price incorporates all relevant information including past spot and futures prices. The investigation of market efficiency through the spot-futures relationship is often approached in a cointegrating framework. Market efficiency implies cointegration between the same factors that determine the future spot price are reflected in the current futures price, so the two price series should not drift apart. Cointegration between the spot and futures price series of commodities can therefore be used to test for this unbiasedness. Such studies in metals markets include, to list few, nonferrous metal (Chowdhury (1991)), gold-silver spot and futures spread (Wahab et al. (1994)).

Third, let us note that cointegration is not the only econometric tool designed to capture equilibrium relationships in commodity markets. Four types of methodologies are currently being developed in the academic literature:

1. Dynamic correlation models have been proposed, some following Engle (2002)'s Dynamic Conditional Correlation (DCC) model and its extensions. These approaches aim at measuring how correlations across markets can move together – which is usually regarded as a measure of contagion. Examples of this kind can be found in the models by Karolyi and Stulz (1996), Longin and Solnik (2001), Ang and Chen (2002) and Forbes and Rigobon (2002).

2. Copulas – a more general dependence structure than the underlying Gaussian model behind the dynamic correlation models – went through an increasing attention, as presented in Patton (2006), Jondeau and Rockinger (2006) and Rodriguez (2007) for standard assets and in Reboredo (2011) and Delatte and Lopez (2012) in the case of commodities. However, beyond their ability to measure tail dependence – i.e. joint extreme events – turning them into a dynamic measure of dependence turned out to be difficult.

3. Multivariate Markov Switching model also encompass in a certain way the measurement of cross time series dependence, as presented in Khalifa et al. (2012). Their ability to produce switches between two types of dependence structures led to interesting findings in

financial markets (e.g. Ielpo, 2012). Their numerical complexity when increasing the number of regimes is however a massive drag to their use to measure cross-market dynamics. This burden to estimate sophisticated specifications of such models in the case of cross-commodity measurement limitates their use.

4. Finally, Diebold and Yilmaz (2012) have presented a new approach based on Vector Autoregressive models (VAR), a time series model that is now widely spread and understood in the academic literature. They propose to measure cross asset volatility spillovers through the historical cross-series mean reversion parameters. Their approach has been used in Yilmaz (2010), Kocenda et al. (2011) and da Fonseca and Gottschalk (2012) for example. The apparent simplicity and efficiency of the estimates obtained turn this approach into a very promising one.

Nonetheless, in this article, we prefer to focus solely on the cointegration approach, which requires a strong economic rationale to attempt to relate two (or more) variables together overtime.

This article focuses on the core of the long-run relationship within metals commodities. It employs unit root tests, cointegration and VECM methodologies to analyze daily data over the 1993-2011 period. The rest of the paper is structured as follows. Section 2 reviews the extant literature.Section 3 details the data used. Section 4and 5 develop cointegration analyses applied to, respectively, industrial and precious metals. Section 6 conludes. The Appendix contains a review of the cointegration methodology, with and without structural breaks detection.

2. Literature Review

In this section, we review some of the main results from previous academic literature on the topic of cointegration in metals markets. The distinction between industrial and precious metals is necessary, because it reflects industry dynamics. These two kinds of metals are not used for the same purposes, and it would not be meaningful to study them together in a cointegrated system, whose rationale is to link series in the long-term because they share common trends. Regarding this classification and sub-grouping of commodity metals, we can relate to various indexes that exist in the industry (e.g. Reuters/ Jefferies Commodity Research Bureau, or Goldman Sachs commodity sub-indices).

2.1 Industrial metals

• MacDonald and Taylor (1988) investigate the cointegration between the prices of tin, zinc and lead on the London Metal Exchange (LME) during 1976-1985. The authors report overwhelming evidence of no cointegration for any of the metal prices considered in their study.

• By using data from 1900 to 1986, von Hagen (1989) shows that primary and manufactured goods prices are cointegrated, i.e. that the hypothesis of stationarity for the long run relative price movements cannot be rejected.

• Franses and Kofman (1991) consider equilibrium relationships among aluminum, copper, lead, nickel and zinc prices on the LME in 1981. The authors are able to find one cointegrating relationship, with the copper price reacting rapidly to disequilibrium errors (and in this respect being less exogenous than other metals).

• Labson and Crompton (1993) examine whether a long run stationary equilibrium relationship holds between aggregate income and consumption of various primary industrial metals, e.g. aluminum, copper, lead, steel, tin, zinc, during 1960-1992. They find little evidence to support the presence of a long run equilibrium relationship between income and metals consumption.

• Chen and Lin (2004) examine the dynamic relation between the LME lead price and its possible predictors during 1964-1995. Their results highlight the presence of one cointegration relationship between lead prices, inventories, and UK treasury bill rates.

• Cerda (2007) identifies a cointegration relationship between the price of copper, exchange rates and wholesale price indices during 1994-2003. The author identifies that the demand from large economic blocs (especially Asia) significantly affects the price of copper.

The main insights from these studies have been summarized in Table 1.

Table 1. Industrial Metals: Cointegrating Relationships

Authors	Period	Cointegration Relationship	SS	SB
MacDonald and Taylor (1988)	1976-1985	Tin $\varnothing$ Zinc $\varnothing$ Lead	No	No
von Hagen (1989)	1900-1986	PrimaryGoods $\leftrightarrow$ ManufacturedGoods	No	No
Franses and Kofman (1991)	1981	Aluminum $\leftrightarrow$ Copper $\leftrightarrow$ Lead $\leftrightarrow$ Nickel $\leftrightarrow$ Zinc	No	No
Labson and Crompton (1993)	1960-1992	Aluminum $\varnothing$ Copper $\varnothing$ Lead $\varnothing$ Steel $\varnothing$ Tin $\varnothing\leftrightarrow$ Zinc	No	No
Chen and Lin (2004)	1964-1995	Lead $\leftrightarrow$ Inventories $\leftrightarrow$ UK Treasury Bill	No	No
Cerda (2007)	1994-2003	Copper $\leftrightarrow$ Exchange Rate $\leftrightarrow$ Price Indices	No	No

Note: $\leftrightarrow$ indicates the presence of a cointegration relationship. $\varnothing$ indicates the absence of a cointegration relationship. SS stands for 'Sub Sample' analysis in the paper considered. SB stands for 'Structural Break' analysis in the paper considered.

It seems that the academic research has been focusing little attention on the cointegration relationships between industrial metals, due to the predominant view that each metal market is very dependent upon its own supply and demand fundamentals.

Nonetheless, we wish to test whether this market practitioners' view is valid, and pursue our analysis in Section 4 by investigating the cointegration issue between aluminum, copper, nickel, zinc, and lead prices during 1993-2011.

2.2 Precious metals

Next, we explore the main findings from the academic literature on cointegration among precious metals. Our review is carried out by analyzing together *(i)* gold and silver prices, and *(ii)* gold, palladium and platinum prices.

2.2.1 Gold and silver prices

• Wahab et al. (1994) first identified a cointegrating relationship between gold and silver during 1982-1992. Escribano and Granger (1998) pursue this analysis based on monthly data during 1971-1990, and identify the influence of a large bubble from 1979:9 to 1980:3. The authors demonstrate that the bubble period had a lasting influence on cointegration, on the short run dynamics, and possibly on the nonlinearity of the relationship.

• Ciner (2001) examines the long run trend between the prices of gold and silver futures contracts traded on the Tokyo Commodity Exchange (TOCOM) during 1992-1998. Contrary to previous studies, the cointegration tests do not support a stable long-run relationship between gold and silver prices in the futures markets. This finding indicates that these two markets should be approached as separate markets, and that changes in the gold-to-silver ratio should not be used to predict prices in the future. Another implication is that these two markets should not be regarded as substitutes to hedge against similar types of risks. This view is consistent with the understanding that these two commodities have different economic uses, and consequently that they are affected by different economic fundamentals.

• Lucey and Tully (2006) examine the dynamic relationship between gold and silver during 1978-2002. A stable, long run relationship exists between gold and silver returns over the period examined. The authors conclude that the stable relationship between gold and silver found to prevail historically appears to have continued during their sample period. There are also significant periods during which the cointegrating relationship is weakened or broken. This may indicate that the results of Ciner (2001) are driven by the period under analysis. For portfolio managers and investors, the overall message is that while gold and silver, in general, offer little diversification advantages when included together in a portfolio, this relationship is not stable. Thus, there may be potential to include both precious metals at certain times.

2.2.2 Gold, palladium and platinum prices

• Kearney and Lombra (2009) attempt to explain the behavior of gold prices relative to platinum prices during 1985-2006. They do not reject the null of no cointegration between gold and platinum prices over the full sample period. However, the authors detect a sub-sample from 1996 through 2006 during which the two time series seem to behave differently. Kearney and Lombra (2009) uncover that forward sales are negatively related to gold prices and equilibrium errors during this period. Hence, the increase in forward sales in the 1990s adversely affected gold prices and therefore altered the return on gold, its relationship with platinum prices and, by extension, other assets considered to be close substitutes in investors' portfolios.

• Tsuchiya (2010) investigates whether the TOCOM gold, silver, palladium and platinum futures prices move independently in the long run by relying on the cointegration approach during 2002-2010. The author finds that the prices of the gold, palladium, platinum and silver futures contracts move independently, i.e. that there is no long term relationship among them.

These main results from academic research are conveniently summarized in Table 2.

Table 2. Precious Metals: Cointegrating Relationships

Authors	Period	Cointegration Relationship	SS	SB
Gold and silverprices				
Wahab et al. (1994)	1982-1992	Gold ↔ Silver	No	No
Escribano and Granger (1998)	1971-1990	Gold ↔ Silver	Yes	Yes
Ciner (2001)	1992-1998	Gold ∅ Silver	No	No
Lucey and Tully (2006)	1978-2002	Gold ↔ Silver	Yes	No
Gold, palladium and platinum prices				
Kearney and Lombra (2009)	1985-2006	Gold ↔ Platinum	Yes	No
Tsuchiya (2010)	2002-2010	Gold ∅ Silver ∅ Palladium ∅ Platinum	No	No

Note: ↔ indicates the presence of a cointegration relationship. ∅ indicates the absence of a cointegration relationship. SS stands for ‘Sub Sample’ analysis in the paper considered. SB stands for ‘Structural Break’ analysis in the paper considered.

Overall, we notice that meaningful equilibrium relationships can be detected in precious metals markets. Most studies agree on the fact that gold and silver on the one hand, gold, palladium and platinum on the other hand are characterized by a common evolution in the long term (despite the occurrence of time periods during which the specific supply and demand fundamentals of each market may cause the price series to diverge).

In the next section, we present a preliminary analysis of the data used in the article.

3. Dataset and unit root tests results

Table 3 presents descriptive statistics on the metals prices used in this article, i.e. gold, silver, platinum, aluminium, copper, zinc, nickel and lead. All the data comes from Bloomberg in daily frequency. The dataset starts in 1993 and ends in 2011. More precisely, concerning the specific time series retained in our article, Gold is the Commodity Exchange (COMEX) Gold futures price traded in US Dollar per troy ounce, Silver is the COMEX Silver futures price traded in US Dollar per troy ounce, Platinum is the COMEX Platinum futures price in US Dollar per troy ounce, Aluminum is the London Metal Exchange (LME) futures price traded in US Dollar per tonne, Copper is the COMEX Copper futures price traded in US Dollar per pound, Nickel is the LME Nickel futures price traded in US Dollar per tonne, Zinc is the LME futures price traded in US Dollar per tonne, and Lead is the LME futures price traded in US Dollar per tonne. The descriptive statistics concern the variables transformed to log-returns. We can notice departure from normality with excess kurtosis and a skewness coefficient different from 3. This comment is further confirmed by the value of the Jarque Bera test statistic. In this paper, the focus in on the determination of long term relationships. That is why we do not test explicitly for the presence of GARCH-type errors.

Table 3. Descriptive statistics for metals prices [1993-2011]

	Min	Max	Mean	Std. Dev.	Skew.	Kurt.	JB
Gold	5.5324	7.5483	6.2847	0.5795	0.5762	5.1608	268.6936
Silver	1.2733	3.8803	2.1765	0.6662	0.7072	8.0458	287.2413
Platinium	5.8141	7.7189	6.6733	0.5767	0.0201	6.3994	241.9912
Aluminium	6.9475	8.1068	7.5065	0.2654	0.2760	2.6170	160.6329
Copper	7.2145	9.2262	8.1567	0.6461	0.1949	2.6544	291.6874
Nickel	8.2441	10.8513	9.4000	0.6158	0.1708	3.0440	141.9007
Zinc	6.6107	8.4152	7.2859	0.4437	0.5765	2.4534	219.4487
Lead	5.9269	8.2662	6.8975	0.6528	0.3162	3.0693	261.9778

Note: The number of observations is equal to 2,757. Std. Dev. stands for Standard Deviation, Kurt. for Kurtosis, Skew. for Skewness, and JB for the Jarque Bera test statistic.

The time series used are shown in logarithm form in Figure 1 for industrial metals, and in Figure 2 for precious metals. While the prices of precious metals seem to rise together in the long run, industrial metals display an homogeneous behavior across the time period, with similar price movements during periods of expansion / recession. Hence, by means of this preliminary visual inspection, we validate intuitively the need to resort to cointegration to analyze in more details the behavior of the respective groups of metals commodities included in this article.

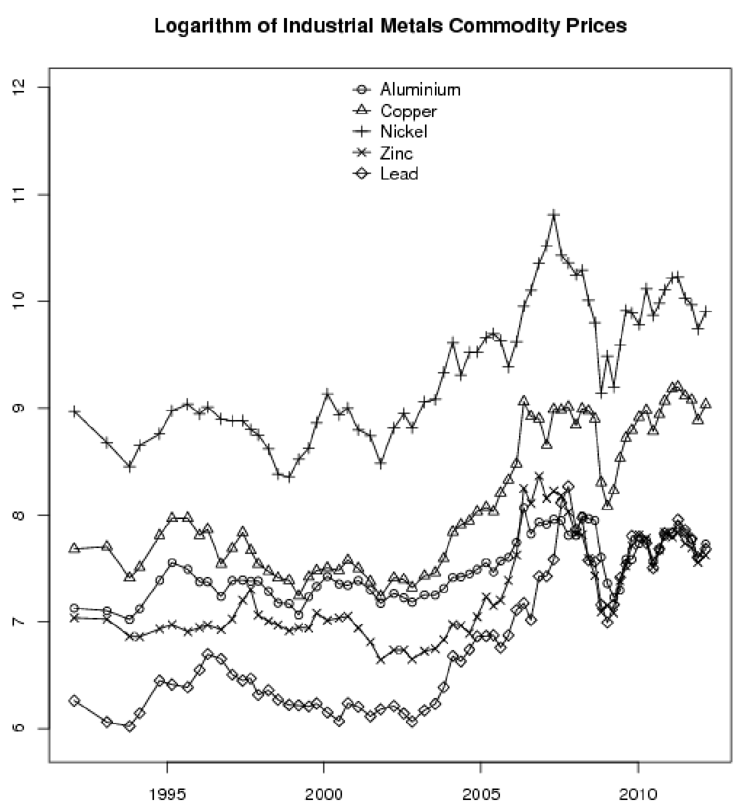

Figure 1. Logarithm of time series for industrial metals

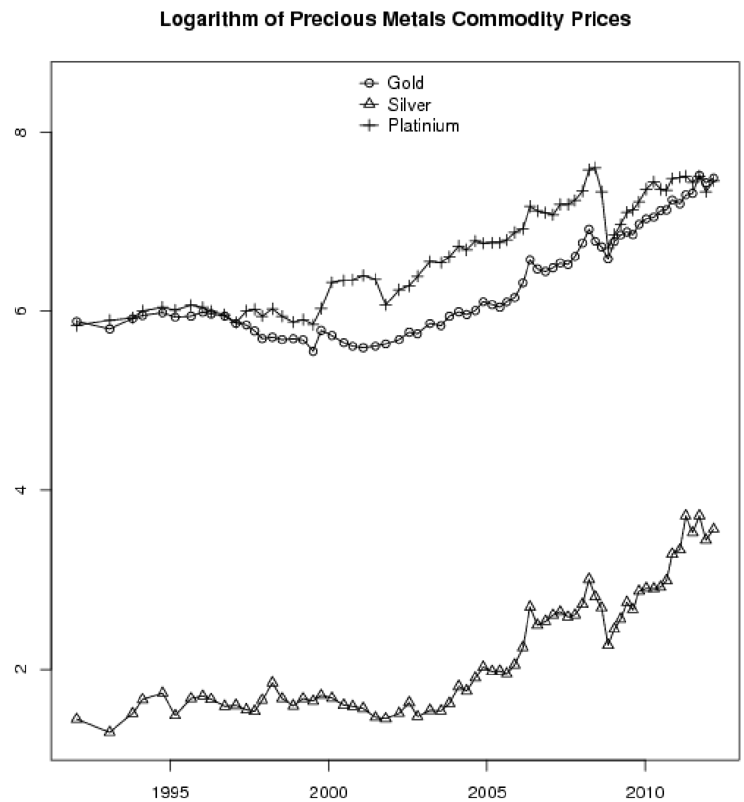

Figure 2. Logarithm of time series for precious metals

Table 4. Unit root test results for metals prices

	ADF None	ADF Drift	ADF Trend	PP Constant	PP Trend	KPSS
Gold	-37.1056	-37.1925	-37.2477	-52.9168	-52.9698	0.0524
Silver	-37.3650	-37.4052	-37.4082	-55.1757	-55.1768	0.0279
Platinium	-38.4727	-38.5155	-38.5097	-53.1674	-53.1589	0.0525
Aluminium	-37.5246	-37.5248	-37.5216	-53.6836	-53.6768	0.0499
Copper	-37.9764	-37.9995	-38.0015	-54.2472	-54.2451	0.0973
Nickel	-36.5636	-36.5627	-36.5561	-52.6028	-52.5934	0.0804
Zinc	-37.5732	-37.5711	-37.5647	-54.2649	-54.2554	0.0906
Lead	-36.8914	-36.9064	-36.9001	-50.4940	-50.4850	0.0709
WTI	-38.6010	-38.6222	-38.6159	-54.7841	-54.7748	0.0434

Note: Test statistics are given. ADF stands for the Augmented Dickey-Fuller unit root test, PP for the Phillips-Perron unit roor test, and KPSS for the Kwiatkowski Phillips Schmidt Shin unit root test. Corresponding critical values (at 5% level) can be found in Greene (2011): -1.9409 for ADF None, -2.8623 for ADF Drift, -3.4114 for ADF Trend, -2.8623 for PP Constant, -3.4114 for PP Trend, and 0.4630 for KPSS.

Table 4 reports the usual unit root tests (ADF, PP, KPSS) results for the prices of metals markets. These tests are meant to check formally the stationarity of the time series under consideration, in addition to the preliminary investigation of the plot for each time series. The results reproduced concern the *log-returns*. For the ADF and PP tests, the null hypothesis is that the variable contains a unit root, and the alternative is that the variable was generated by a stationary process. The test statistics reproduced are far smaller than the corresponding critical values (which can be found in Greene (2011) for instance, and mentioned at the bottom of the table). This leads us toreject the null hypothesis. Hence, we can safely conclude that all time series transformed to log-returns are stationary. For the KPSS test, we consider the null hypothesis of stationarity against the alternative of a unit root. The test statistics are smaller than the critical values (see again Greene (2011)), which leads us to accept the null hypothesis of stationarity. Hence, we verify that all series are integrated of the same order (*I(1)*), which is a pre-condition for cointegration.

4. Assessing the cross-commodity relationships in industrial metals markets

Cointegration between the main industrial metals can be of interest to researchers and practitioners, since some metals have a substitutive character, while others may be seen as complementary.

4.1 Results of Granger-causality tests for industrial metals

Table 5 displays the results of the pairwise Granger causality tests among industrial metals (with one lag for the sake of simplicity). Aluminum is found to Granger cause Lead. Copper causes in the Granger sense Nickel and Lead (at the 5% level). Nickel does not display significant Granger causalities. Zinc is found to Granger cause Aluminum (at the 10% level). Finally, Lead displays a significant Granger causality with Zinc. Thus, there may be some inter-relationships at stake between industrial metals. That is why we resort to a cointegration analysis in the next section.

Table 5. Pairwise Granger causality tests for industrial metals

From	To	p-value	F-statistic
Aluminium	Copper	0.5118	0.4305
Aluminium	Nickel	0.1989	1.6509
Aluminium	Zinc	0.6568	0.1975
Aluminium	Lead	0.0318	4.6104
Copper	Aluminium	0.2980	1.0834
Copper	Nickel	0.0414	4.1631
Copper	Zinc	0.4157	0.6624
Copper	Lead	0.0534	3.7332
Nickel	Aluminium	0.3498	0.8743
Nickel	Copper	0.7418	0.1086
Nickel	Zinc	0.7688	0.0864
Nickel	Lead	0.2040	1.6138
Zinc	Aluminium	0.0745	3.1819
Zinc	Copper	0.2179	1.5187
Zinc	Nickel	0.1109	2.5426
Zinc	Lead	0.1287	2.3084
Lead	Aluminium	0.8061	0.0603
Lead	Copper	0.5445	0.3673
Lead	Nickel	0.7522	0.0997
Lead	Zinc	0.0077	7.1105

Note: The p-value and the F-statistic of the pairwise Granger causality tests between X and Y are given. The null hypothesis is that X does not Granger cause Y.

4.2 Cointegration analyses for industrial metals

Concerning industrial metals, the main results from our cointegration analyses are summarized in Table 6.

Table 6. Cointegration Analyses of Industrial Metals: Summary of the Main Results

Period	Cointegration Relationship	SB
1993-2011	Aluminum ∅ Copper ∅ Nickel ∅ Zinc ∅ Lead	No
1993-2000	Aluminum ∅ Copper ∅ Nickel ∅ Zinc ∅ Lead	No
2000-2011	Aluminum ∅ Copper ∅ Nickel ∅ Zinc ∅ Lead	No
1993-2011	Aluminum ↔ Copper ↔ Nickel ↔ Zinc ↔ Lead	Yes
1993-2011	Zinc ∅ Lead	No
1993-2000	Zinc ∅ Lead	No
2000-2011	Zinc ∅ Lead	No
1993-2011	Zinc ↔ Lead	Yes

Note: ↔ indicates the presence of a cointegration relationship. ∅ indicates the absence of a cointegration relationship. SB stands for 'Structural Break' analysis.

Note that we did not wish to replicate here the results by Chen and Lin (2004) or Cerda (2007), who made use of Bonds ad Exchange Rates data. This type of research is left for further developments by using that kind of data. Here, we focus more specifically on the relationships within metals commodities.

In what follows, sub-sample dates are detected numerically (i.e. not exogenously set by the user) by resorting to Lutkephol et al.'s (2004) cointegration tests with one structural break.

4.2.1 Aluminum, Copper, Nickel, Zinc and Lead

Our first cointegration exercise concerns the five time series of industrial metals contained in our database, i.e. Aluminum, Copper, Nickel, Zinc and Lead. While no cointegration relationship could be found during either the full period or the corresponding sub-periods, Table 7 reports the results obtained when allowing for the presence of one structural break.

Table 7. Lütkepohl et al. (2004) Cointegration Test Results with Structural Break for Aluminum, Copper, Nickel, Zinc and Lead

1993-2011	Max. Eigen.	10%	5%	1%
$r \leq 4$	5.65	5.42	6.79	10.04
$r \leq 3$	14.16	13.78	15.83	19.85
$r \leq 2$	29.35	25.93	28.45	33.76
$r \leq 1$	53.63	42.08	45.2	51.6
$r = 0$	82.02	61.92	65.66	73.12

From this Table, we learn that the rank of the cointegration r is at least equal to 1, i.e. we are able to find at least one stationary combination between the variables of interest in the industrial metals category. In the VECM analysis, the lag length is set to oneby letting the AIC and BIC criteria determine the lag length of the model.

The estimation of the VECM returns the results given in Table 8. Four out of five error correction terms turn out to be negative. Three ECTs are significant at the 1% level (as marked by ***): Aluminum, Nickel and Zinc. Therefore, the results are quite satisfactory, and the VECM can be considered as valid. When interpreting the size of the coefficients of the ECTs, we remark that they are roughly equal (around -0.008), but that the Aluminum stands out as having the highest significance level. Hence, in this system composed of five industrial metals, any short term deviation from the long run equilibrium can be corrected by Aluminum as the driving force, followed by Nickel and Zinc. It is very interesting to be able to find such a strong error correction mechanism between these markets, which are often judged as being separate markets operating based on their own demand and supply fundamentals, and whose trading is predominantly based on expert opinion (as in the LME). Contrary to this view, we highlight that there is a common variation trend among these industrial metals, which cannot be priced purely independently.

Table 8. VECM Results with Structural Break (1993-2011) for Aluminum, Copper, Nickel, Zinc and Lead

Error Correction Term					
Aluminum	1				
Copper	-0.288				
Nickel	0.318				
Zinc	0.169				
Lead	-0.898				
VECM	Δ Aluminum	Δ Copper	Δ Nickel	Δ Zinc	Δ Lead
ECT	-0.008***	-0.004	-0.009***	-0.008***	0.003
(t.stat)	(-3.73)	(-1.4)	(-2.36)	(-2.79)	(0.96)
Δ Aluminum(-1)	0.014	0.032	0.004	0.031	-0.053
(t.stat)	(0.52)	(0.89)	(0.09)	(0.83)	(-1.2)
Δ Copper(-1)	-0.015	-0.029	-0.071	-0.057	-0.028
(t.stat)	(-0.66)	(-0.98)	(-1.85)	(-1.88)	(-0.77)
Δ Nickel(-1)	-0.002	-0.002	0.032	-0.006	0.001
(t.stat)	(-0.12)	(-0.13)	(1.36)	(-0.31)	(0.03)
Δ Zinc(-1)	-0.032	-0.044	-0.028	-0.054	0.001
(t.stat)	(-1.55)	(-1.59)	(-0.77)	(-1.87)	(0.04)
Δ Lead(-1)	0.006	0.013	0.018	0.063	0.055
(t.stat)	(0.38)	(0.61)	(0.65)	(2.88)	(2.12)

The cointegration relationship pictured in Figure 3 reveals the presence of a structural break on May 14, 2007. Before and after that date, the cointegration relationship appears stationary during each of the two regimes. These results confirm the findings by Franses and Kofman (1991), and are more optimistic than the ones by Labson and Crompton (1993). These latter authors considered a subset of industrial metals composed of Aluminum, Copper, Lead, Steel, Tin and Zinc, but failed to identify any cointegration relationship. To conclude this section, the cointegration relationship between industrial metals seems robust, once we control for the presence of a structural break. The driving force behind these common trends could be understood as being economic activity, which fosters the need for industrial metals as an input to production goods.

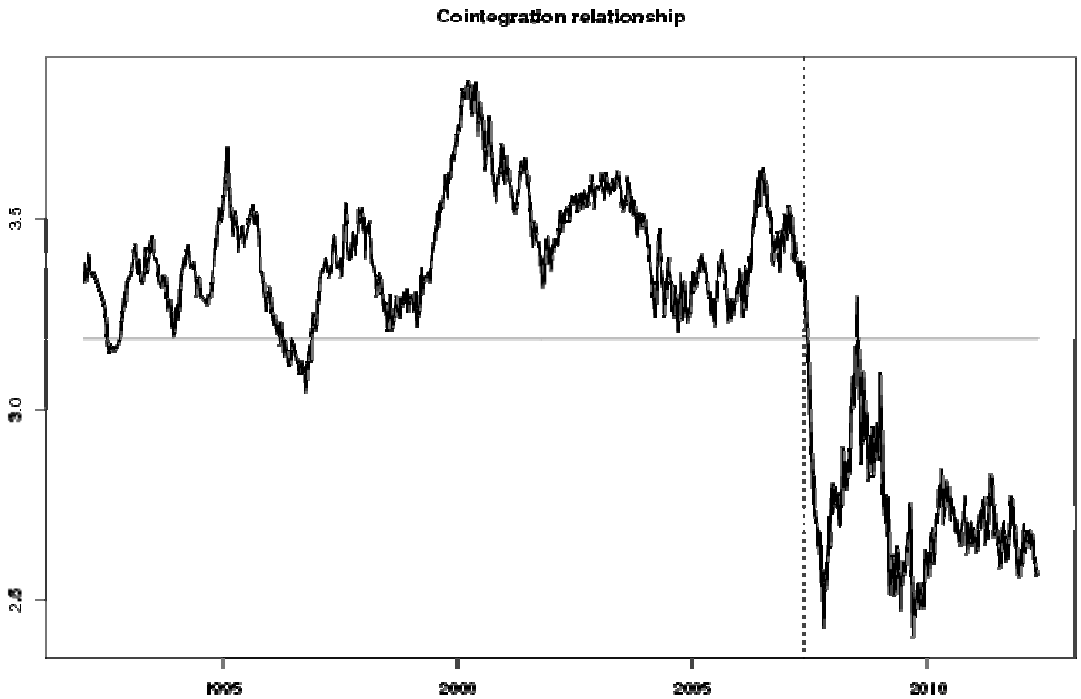

Figure 3. Cointegration relationship for the 1993-2011 full period between Aluminum, Copper, Nickel, Zinc and Lead

4.2.2 Zinc and Lead

In our second cointegration exercise with industrial metals, we choose to focus on the pair of commodities composed of Zinc and Lead. Similarly to the previous case, we could identify one cointegration relationship only during the full period and by modeling explicitly one structural break.

Table 9. Lütkepohl et al. (2004) Cointegration Test Results with Structural Break for Zinc and Lead

1993-2011	Max. Eigen.	10%	5%	1%
$r \leq 1$	4.02	5.42	6.79	10.04
$r = 0$	15.64	13.78	15.83	19.85

These results are shown in Table 9: at the 10% level, we reject the null hypothesis that r =0 in favor of one linear stationary combination of the two time series considered here.

However, the VECM estimation results are not as satisfactory as in the preceding case: none of the error correction terms is significantly negative in Table 10.

The inspection of Figure 4 reveals that – before and after the structural break dated on November 28, 2006 – the two regimes are not stationary and impacted by ample fluctuations coming from either Zinc or Lead. Therefore, our results confirm the findings by MacDonald and Taylor (1988) who could not identify a robust cointegration relationship between Zinc and Lead. Note that their analysis included also Tin.

Table 10. VECM Results with Structural Break (1993-2011) for Zinc and Lead

Error Correction Term		
Zinc	1	
Lead	-2.10	
VECM	Δ Zinc	Δ Lead
ECT	0	0.003***
(t.stat)	(-0.14)	(2.44)
Δ Zinc(-1)	-0.081	-0.051
(t.stat)	(-3.15)	(-1.66)
Δ Lead(-1)	0.059	0.048
(t.stat)	(2.74)	(1.87)

Figure 4. Cointegration relationship with structural break between Zinc and Lead

4.3 Summary and discussion on cross-commodity relationships in industrial metals markets

To sum up the main findings from our cointegration analyses applied to the case of industrial metals, we find in line with Franses and Kofman (1991) that there exist common variations between Aluminum, Copper, Nickel, Zinc and Lead which gear toward a linear stationary combination in the long run. This result is obtained by including one structural break in the model. However, the same conclusion could not be reached for the pair Zinc and Lead.

Overall, these results are quite optimistic for two main reasons. First, unlike most of the wisdom coming from trading experts in the LME, industrial metals need not necessarily be considered as separate markets. Second, these findings teach us that industrial metals could be traded based on a common information signal. This signal could be the extent to which an economy is booming (or entering recession), which is reflected in the need of industrial metals as necessary inputs to production processes.

In what follows, we turn our attention to the case of precious metals.

5. Assessing the cross-commodity relationships in precious metals markets

Gold and silver have historically been seen as close substitutes for one another, both being precious metals that can be used to back currency and both having been used as currency. Moreover, Adrangi et al. (2003) document that these metals can play a useful role in diversifying risk, as well as being an attractive investment in their own right. Therefore, one might expect that gold and silver prices share similar dynamics.

However, there are also economic fundamentals that may act to drive the prices of gold and silver apart. While both are used extensively in industrial processes, there are significant differences between these uses. Silver is extremely reflective, a good conductor of electricity, and has extensive use in optics and photography. Gold's industrial uses are fewer, with the majority of demand coming from the jewellery and dental markets (about 50% of the newly mined gold is used for jewellery and 40% for investment purposes), as well as being driven by the demand from Central Banks.

Besides gold and silver, palladium and platinum are considered as attractive assets for portfolio investment, especially during times of rising inflation and global economic and political instability. Their price fluctuations seem to follow closely the price path of gold over the last two decades (Kearney and Lombra (2009)).

Gold, palladium and platinum can follow different price paths during specific time periods. Their respective price fundamentals can differ widely, as industrial use occupies approximately two thirds of the total demand for palladium and platinum. Gold can also be traded independently as a refuge for value during periods of bearish markets.

Given their relative scarcity and high economic value, gold, silver, palladium and platinum are therefore considered together as precious metals in cointegration analyses.

In what follows, we proceed with our own cointegration analysis between gold, silver, palladium and platinum.

5.1 Results of Granger-causality tests for precious metals

Table 11 reports the results of pairwise Granger causality tests (with one lag) among precious metals. It is difficult to infer causal relationship from this Table, but it can be seen as a guide for the cointegration analysis. For instance, we note that gold and silver do not necessarily cause each other (at the 5% rejection rate), while the relationship between gold and platinum goes in both directions.

Table 11. Pairwise Granger causality tests for precious metals

From	To	p-value	F-statistic
Gold	Silver	0.3345	0.9314
Gold	Platinium	0.0216	5.2793
Gold	WTI	0.8239	0.0495
Silver	Gold	0.0725	3.2274
Silver	Platinium	0.0165	5.7555
Platinium	Gold	0.0098	6.6769
Platinium	Silver	0.7209	0.1276

Note: The p-value and the F-statistic of the pairwise Granger causality tests between X and Y are given with one lag. The null hypothesis is that X does not Granger cause Y.

5.2 Cointegration analyses for precious metals

The results are summarized in Table 12.

Table 12. Cointegration Analyses of Precious Metals: Summary of the Main Results

Period	Cointegration Relationship	SB
1993-2011	Gold $\varnothing$ Silver	No
1993-2000	Gold $\varnothing$ Silver	No
2000-2011	Gold $\varnothing$ Silver	No
1993-2011	Gold $\leftrightarrow$ Silver	Yes
1993-2011	Gold $\varnothing$ Platinum	No
1993-2000	Gold $\varnothing$ Platinum	No
2000-2011	Gold $\leftrightarrow$ Platinum	No
1993-2011	Gold $\leftrightarrow$ Platinum	Yes
1993-2011	Gold $\varnothing$ Silver $\varnothing$ Platinum	No
1993-2000	Gold $\varnothing$ Silver $\varnothing$ Platinum	No
2000-2011	Gold $\leftrightarrow$ Silver $\leftrightarrow$ Platinum	No
1993-2011	Gold $\leftrightarrow$ Silver $\leftrightarrow$ Platinum	Yes

Note: $\leftrightarrow$ indicates the presence of a cointegration relationship. $\varnothing$ indicates the absence of a cointegration relationship. SB stands for 'Structural Break' analysis.

5.2.1 Gold and Silver

Let us start with the most frequently studied pair of commodities in the category of precious metals, i.e. the relationship between Gold and Silver. Table 12 tells us that only one cointegrating relationship can be detected during the full period with the occurrence of a structural break.

Table 13. Lütkepohl et al. (2004) Cointegration Test Results with Structural Break for Gold and Silver

1993-2011	Max. Eigen.	10%	5%	1%
$r \leq 1$	9.23	5.42	6.79	10.04
r = 0	23.39	13.78	15.83	19.85

In Table 13, we verify indeed that the null of no cointegration can be rejected at the 1% level in favor of the alternative hypothesis that there exists at least one cointegrating relationship between Gold and Silver during 1993-2011.

Table 14. VECM Results with Structural Break (1993-2011) for Gold and Silver

Error Correction Term		
Gold	1	
Silver	-0.508	
VECM	Δ Gold	Δ Silver
ECT	-0.008**	0.003
(t.stat)	(-2.06)	(0.61)
Δ Gold(-1)	0.024	0.054
(t.stat)	(0.85)	(1.36)
Δ Silver(-1)	-0.027	-0.07
(t.stat)	(-1.35)	(-2.48)

The estimation of the VECM, shown in Table 14, reveals that only the error correction term for Gold is negative (equal to -0.008) and statistically significantat the 5% level (as marked by **). Hence, we can infer that Gold exhibits the strongest error-correction effect among the two commodities. This result is not surprising given the common knowledge that Gold constitutes the safest precious metal (in terms of returns on investment) in its category.

Regarding the cointegration relationship, represented in Figure 5, it is readily observable that it is not stationary after the structural break on November 14, 1996. Indeed, past that date, the cointegrating relationship seems to diverge again from 2008 onwards, probably indicating the presence of more than one break. We cautiously conclude that there seems to be an unstable cointegrating relationship between Gold and Silver, which may be due to diverging market fundamentals between the two commodities. Silver has more applications in the industry than Gold for instance, hence making the two time series periodically departing from each other. Our results are similar to Ciner (2001), but contradict the previous findings by Wahab et al. (1994), Escribano and Granger (1998), and Lucey and Tully (2006).

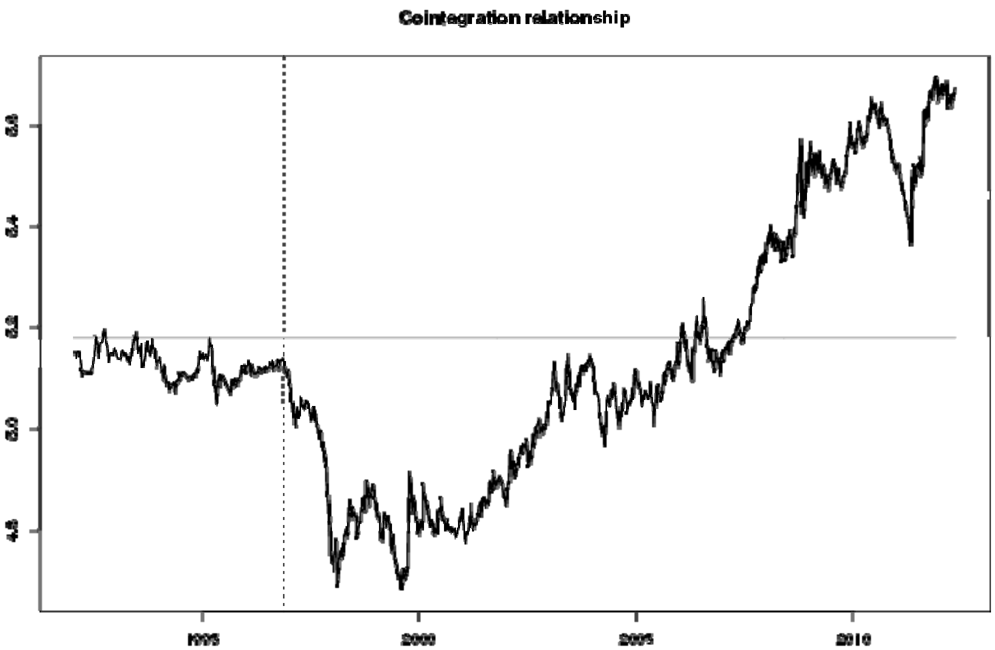

Figure 5. Cointegration relationship with structural break between Gold and Silver

5.2.2 Gold and Platinum

The second pair of precious metals considered here is composed of Gold and Platinum. For the first time in our cointegration analyses, we are able to detect a valid cointegrating relationship during a sub-period, namely 2000-2011.

Table 15. Cointegration test for the 2000-2011 sub period, *without* structural break, between Gold and Platinum

2000-2011	Max. Eigen.	10%	5%	1%
r ≤ 1	3.76	10.49	12.25	16.26
r = 0	21.57	16.85	18.96	23.65

Table 15 provides the results of the linear Johansen cointegration test during the recent period, i.e. 2000-2011. At the 5% level, we can assess that the rank of the cointegration r is at least equal to 1.

The estimation of the VECM in Table 16 provides only half of the expected results, since only the error correction term for Gold is negative and highly significant at the 1% level (as marked by ***). Similarly to the preceding case, Gold possesses the strongest error-correction effect towards the long-run equilibrium in that system as well. Any short term departures from the long run equilibrium will be corrected by the variations of the Gold price to push back towards the fundamental value between these two precious metals.

Table 16. VECM results for the 2000-2011 sub period, *without* structural break, between Gold and Platinum

Error Correction Term		
Gold	1	
Platinum	-0.288	
Trend	-0.001	
VECM	Δ Gold	Δ Platinum
ECT	-0.027***	0.001
(t.stat)	(-3.44)	(0.11)
Intercept	0.112	-0.004
(t.stat)	(3.47)	(-0.1)
Δ Gold(-1)	0.032	0.032
(t.stat)	(0.91)	(0.75)
Δ Platinum(-1)	-0.089	0.009
(t.stat)	(-3.02)	(0.25)

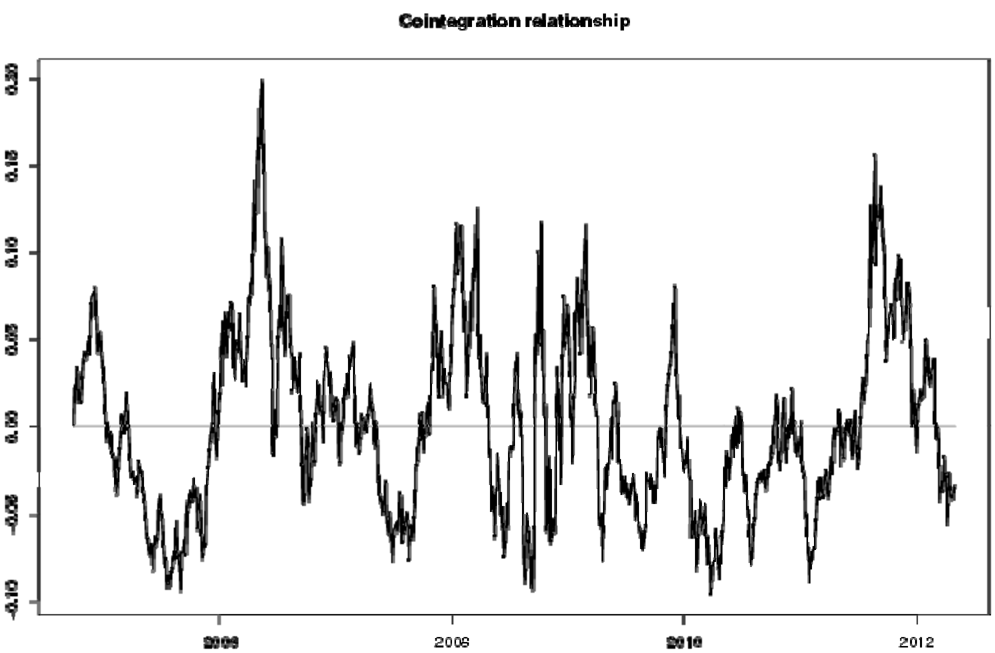

Figure 6. Cointegration relationship for the 2000-2011 sub period, *without* structural break, between Gold and Platinum

The cointegration relationship displayed in Figure 6 appears extremely stable. Thus, we can undoubtedly conclude in favor of the presence of one cointegration relationship between Gold and Platinum. We thereby agree with findings by Kearney and Lombra (2009).

As mentioned in Table 12, note that similar results could be achieved by modeling the cointegrating relationship with one structural break during 1993-2011. In that setting, we identified a break date on July 3, 2008. However, the VECM could not be validated, and Gold and Platinum seemed to obey to different dynamics during 2008-2009 (with the possible spillover effects of the financial crisis). Hence, we do not reproduce these latter results.

5.2.3 Gold, Silver and Platinum

We now proceed with the most complete specification composed of Gold, Silver and Platinum. In Table 12, we notice that the results are roughly similar to the case of Gold and Platinum with the possibility to detect one cointegrating relationship either during the second sub-period (without structural break) or during the full period with the modeling of one break.

Table 17. Cointegration test for the 2000-2011 sub period, *without* structural break, between Gold, Silver and Platinum

2000-2011	Max. Eigen.	10%	5%	1%
r ≤ 2	3.65	10.49	12.25	16.26
r ≤ 1	5.13	16.85	18.96	23.65
r = 0	23.91	23.11	25.54	30.34

In Table 17, we can verify that the rank of the cointegration r is at least equal to 1 (at the 10% level).

Table 18. VECM results for the 2000-2011 sub period, *without* structural break, between Gold, Silver and Platinum

Error Correction Term			
Gold	1		
Silver	-0.063		
Platinum	-0.239		
Trend	-0.001		
VECM	Δ Gold	Δ Silver	Δ Platinum
ECT	-0.031***	-0.046***	0.001
(t.stat)	(-3.6)	(-2.79)	(0.05)
Intercept	0.135	0.196	-0.002
(t.stat)	(3.63)	(2.81)	(-0.04)
Δ Gold(-1)	0.059	0.08	0.052
(t.stat)	(1.27)	(0.91)	(0.92)
Δ Silver(-1)	-0.023	-0.094	-0.017
(t.stat)	(-0.94)	(-1.99)	(-0.55)
Δ Platinum(-1)	-0.079	-0.033	0.015
(t.stat)	(-2.57)	(-0.56)	(0.41)

The VECM results reproduced in Table 18 reveal that two error correction terms are significantly negative for Gold and Silverat the 1% level (as marked by ***). Besides, the magnitude of the coefficients estimated is higher than in the two preceding cases, indicating that there are stronger error correction mechanisms at stake in this system composed of three precious metals. Most of the deviations from the long run stationary equilibrium will be exclusively corrected by Gold and/or Silver in our setting.

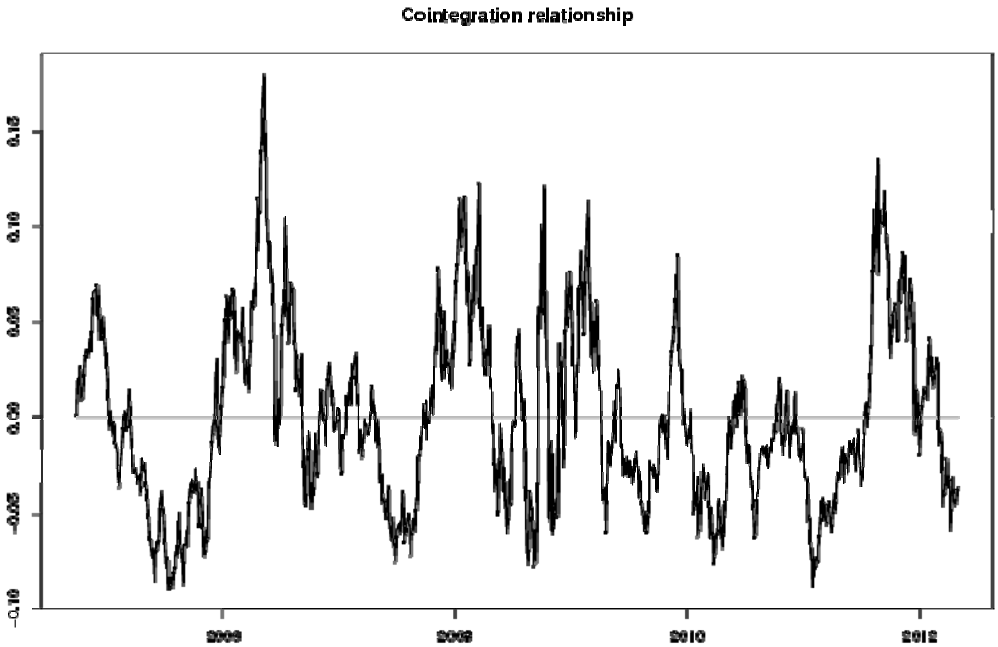

Figure 7. Cointegration relationship for the 2000-2011 sub period, *without* structural break, between Gold, Silver and Platinum

The graph of the cointegrating relationship, given in Figure 7, is stationary. Hence, we have been sucessful in specifying and estimating a cointegration relationship between Gold, Silver and Platinum during 2000-2011. These three precious metals can be considered as having common fundamentals over the period, which trigger their adjustement towards their fundamental value. We disagree on this matter with Tsuchiya (2010), who was unable to identify a valid cointegration relationship between these commodities during 2002-2010 (besides Palladium which is not included in our database).

Note also that Table 12 indicates the presence of another cointegration relationship during 1993-2011 with the occurrence of one structural break on July 15, 2008. In addition to the fact that the VECM could not be validated in that latter case, the cointegrating relationship did not appear stationary during the first regime (i.e. before the break date).

5.3 Summary and discussion on cross-commodity relationships in precious metals markets

The investigation of the cointegration relationships across precious metal has proved to be mainly successful. Indeed, for each of the pair (or more) of commodities investigated, it has been possible to detect at least one cointegration relationship. Therefore, we could broadly conclude that there are more cross-commodity linkages than usually thought by market practitioners.

Interestingly, most of the results hold during the recent 2000-2011 period, during which we have witnessed a commodity boom. Despite the worldwide growth in the demand for many commodities, it appears that some meaningful relationships still exist between precious metals, which share the characteristics of being safe-havens in periods of economic turmoil.

Overall, it cannot be challenged that common variations exist between the variations of the price of precious metals, despite their growing inclusion in production processes for the industry or jewellery. In troubled times, investors can then safely look for precious metals to store value, and their common behavior over the recent period allows to validate this hypothesis.

6. Conclusion

This paper carries out a commodity-by-commodity cointegration analysis of metals markets, namely Gold, Silver, Platinum, Aluminum, Copper, Nickel, Zinc, Lead using daily data for the period 1993-2011.

The primary aim is to determine the long term relationship, if any, between these specific commodities, categorized in two types: precious and industrial metals.

We have attempted in this article to provide a well-documented and exhaustive review of the literature on cross-commodity linkages in metals markets based on the cointegration tool. This topic has been studied in the previous academic literature, but there seemed to lack a central and up-to-date body of knowledge to synthetize this information.

We have divided our database of commodities into two groups: *(i)* industrial metals, and *(ii)* precious metals. We have briefly summarized in the contents of the article the main insights obtained by replicating cointegration analyses for each group.

Compared to the systematic reproduction of the results from previous literature, the interest of resorting to the cointegration technique with the explicit modeling of one structural break has been clearly underlined, compared to linear cointegration tools or sub-period decomposition only. The main conclusion of our empirical work is therefore to indicate that there are more cross-commodity linkages at hand in metals markets than it is usually agreed upon among market practitioners. Even if the relationships at hand are not always stable, they do exist among the various groups of metals commodities investigated in our study.

Across our various econometric models, we have been able to identify sometimes (but not always) meaningful relationships within industrial or precious metals. These pairs or groups of metals seem to share common trends, which might explain why we observe the phenomenon of cointegration over specific periods (or sub-periods) of time. In terms of economic implications, when cointegration is detected, then the long run stationary combination of the time series considered implies that idiosyncratic shocks will be corrected by feedback effects.

The forces at stake in the error correction mechanisms can be related to substitutes and/or complementary relationships between pairs of metals prices. These characteristics are of primary importance for the consumers and producers of the commodity, but also to investment managers and traders who would lose valuable information by ignoring them.

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Appendix: A Primer on Granger-Causality Testing and Cointegration

Granger-Causality Testing

In conjunction with the analysis of the matrix of cross-correlations, the econometrician may resort to Granger causality tests as well. These tests allow to infer causality 'in the Granger sense' between a set of dependent and independent variables selected by the user, and may be useful in econometric modelling prior to the regression analysis. When applied to commodity markets, Granger causality tests will tell us the nature of the inter-relationships between the various markets and categories of commodities.

Recall that a process P_t^1 Granger causes P_t^2 at the order p if, in the linear regression of P_t^2 on lagged prices $P_{t-1}^1,\ldots,P_{t-p}^1,P_{t-1}^2,\ldots,P_{t-p}^2$, at least one of the regression coefficients of P_t^1 on the lagged prices $P_{t-1}^2,\ldots,P_{t-p}^2$ is significantly different from 0. The intuition behind Granger causality is that the information on past prices $P_{t-1}^2,\ldots,P_{t-p}^2$ is relevant to forecast P_t^1 at future time t.

Granger causality is examined using the Granger causality test testing the null hypothesis H_0 that all regression coefficients of P_t^1 on the lagged prices $P_{t-1}^2,\ldots,P_{t-p}^2$ are null. A p-value lower than 0.05 means that H_0 can be rejected (and causality accepted) with 95% confidence level.

Cointegration without Structural Breaks

Cointegration can be seen as a useful econometric tool to decompose the long term trend between pairs (or groups) of variables, and the short-term departures from the trend. In the context of commodity markets, a cointegration relationship will tell us whether a pair (or a group) of individual commodities are tied together in the long run (which means that there exists a strong economic rationale to link these variables in the economic analysis), and to which extent exogenous perturbations from this equilibrium can occur.

Preliminary Conditions

As a pre-requisite condition for cointegration, the time series need to be integrated of the same order. For instance, the econometrician can check, based on standard stationarity tests, that the prices of the raw time series considered are non stationary and integrated of order one ($I(1)$). This amounts to checking that they are difference stationary, i.e. Δx_t^e and $\Delta x_t^{e'}$ are stationaryStationarity is a central concern in time series analysis, which implies that the mean of the variable shall be time invariant (in the weak sense of stationarity). See Hamilton (1996) for further reference..

Before estimating any time series model, it is indeed necessary to test for the stationarity of the dependent variable (and independent variables as well in ARMAX models). Dickey-Fuller (1979) test the nullity of the coefficient α in the following regression:

$$\Delta x_t = x_{t+1} - x_t = \alpha x_t + \beta + e_t \quad \text{... (1)}$$

- if α is significantly negative, then we say that the process x_t has no unit rootThe concept of presence of unit root is similar to the concept of non-stationarity for a given time series variable., or that it is stationary, inducing a mean-reverting behavior for the prices;

- if α is not significantly different from 0, then we say that the process x_t 'has a unit root', inducing a random walk behavior for the prices.

In practice, the Augmented-Dickey-Fuller (1981, ADF) or Phillips-Peron (1988, PP) tests are used rather than Dickey-Fuller. These tests are based on the same principle but correct for potential serial autocorrelation and time trend in Δx_t through a more elaborated regression:

$$\Delta x_t = \sum_{i=1}^{L} \beta_i \Delta x_{t-i} + \alpha x_t + \beta_1 t + \beta_2 + e_t \quad \text{... (2)}$$

The ADF test tests the null hypothesis H_0 that $\alpha = 0$ (the alternative hypothesis H_1 being that $\alpha < 0$) by computing the Ordinary Least Squares (OLS) estimate of α in the previous equation and its t-statistics $\hat{t}$; then, the statistics of the test is the t-statistics $\hat{t}_\alpha$ of coefficient α, which follows under H_0 a known law (studied by Fuller and here denoted Ful). The test computes the p-value p, which is the probability of $Ful \le \hat{t}$ under H_0. If $p < 0.05$, H_0 can be safely rejected and H_1 accepted: we conclude that the series ' x_t has no unit root'. Extensions of these stationarity tests were also developed by Kwiatkowski et al. (1992, KPSS).

Johansen Cointegration Tests

To keep the notations parsimonious, let us consider here the cointegration setting with only two variablesNote however that the Johansen cointegration framework can be generalized to k variables.. As is standard in a linear cointegration exercise, the econometrican needs to check first if the variables are cointegrated, i.e. if β exists such that $R_t = X_t^e - \beta X_t^{e'}$ is stationary. This can be done by performing an OLS regression of X_t^e on $X_t^{e'}$, or more rigorously by using the Johansen cointegration test (Johansen and Juselius (1990), Johansen (1991)).

Let X_t be a vector of N variables, all $I(1)$:

$$X_t = \Phi_1 X_{t-1} + \ldots + \Phi_p X_{t-p} + \varepsilon_t \quad \text{... (3)}$$

with $\varepsilon_t : WGN(0,\Omega)$, WGN denotes the White Gaussian Noise, Ω denotes the variance covariance matrix, and Φ_i $(i = 1,\ldots,p)$ are parameter matrices of size $(N \times N)$.

Under the null H_0, there exists r cointegration relationships between N variables, *i.e.* X_t is cointegrated with rank r.

Note that the Johansen cointegration tests can be performed on the logarithmic transformation of the time series under consideration.

For a financial modeling viewpoint, if we find that commodities are cointegrated, i.e. that there exists a stationary combination of these variables in the long term, the direct implication would be that they share at least one common risk factor in the long term. Hence, their joint analysis can bring fruitful results to the econometrician.

Error-correction model

The next step of the cointegration model consists in describing the dynamics of the variables in terms of the residuals of the long-term relation (Johansen (1988)). We want to introduce an error-correction mechanism on the levels and on the slopes between the variables e and e':

$$\begin{pmatrix} \Delta X_t^e \\ \Delta X_t^{e'} \end{pmatrix} = \begin{pmatrix} \mu_e \\ \mu_{e'} \end{pmatrix} + \sum_{k=1}^{p} \Gamma_k \begin{pmatrix} \Delta X_{t-k}^e \\ \Delta X_{t-k}^{e'} \end{pmatrix} + \begin{pmatrix} \Pi_e \\ \Pi_{e'} \end{pmatrix} R_t + \begin{pmatrix} \varepsilon_t^e \\ \varepsilon_t^{e'} \end{pmatrix} \quad \text{... (4)}$$

where

- e stands for the first variable, and e' stands for the second variable;
- X_t^e is the log price of variable e at time t;
- the 2×1 vector process $\Delta Z_t = \left(\Delta X_t^e = X_{t+1}^e - X_t^e, \Delta X_t^{e'} = X_{t+1}^{e'} - X_t^{e'} \right)$ is the vector of the variables price returns;
- $\mu = (\mu_{X,e}, \mu_{X,e'})$ is the 1×2 vector composed of the constant part of the drifts;
- Γ_k are 2×2 matrices of real valued parameters expressing dependence on lagged returns;
- $(R_t = X_t^e - \beta X_t^{e'})$ is the process composed of the deviations to the long-term relation between the variables log prices;
- Π is a 2×1 vector matrix expressing the sensitivity to the deviations to the long-term relation between the variables prices;

• the residual shocks $(\varepsilon_t^e, \varepsilon_t^{e'})$ are assumed to be i.i.d with a centered bi-variate normal distribution $N(0, \sum)$.

However, by considering a purely linear model, it is possible that the econometrician will either misspecify the model, or ignore a valid cointegration relationship. That is why we detail below the cointegration methodology with an unknown structural break.

Cointegration with Structural Breaks

In this section, we explore the possibility of wrongly accepting a cointegration relationship, when some of the underlying time series are contaminated by a structural break. For instance, sharp deviations from the long-term trend can occur between a group of commodities, which would imply that the cointegration relationship is not valid anymore during specific sub-samples. The structural breakpoint detection allows to take into account these events in the cointegration analysis, instead of simply ignoring them.

We present the procedure for estimating a vector error-correction model (VECM) with a structural shift in the level of the process, as developed by Lütkepohl et al. (2004). By doing so, we draw on the notations by Pfaff (2008).

Framework

Let $\vec{y}_t$ be a $K \times 1$ vector process generated by a constant, a linear trend, and level shift termsNote that Lütkepohl et al. (2004) develop their analysis in the context where $\vec{x}_t$ can be represented as a VAR(p), whose components are at most $I(1)$ and cointegrated with rank r .:

$$\vec{y}_t = \vec{\mu}_0 + \vec{\mu}_1 t + \vec{\delta} d_{t\tau} + \vec{x}_t \qquad \text{... (5)}$$

with $d_{t\tau}$ a dummy variable which takes the value of one when $t \geq \tau$, and zero otherwise. The shift point τ is unknown, and is expressed as a fixed fraction of the sample size:

$$\tau = [T\lambda], \quad 0 < \underline{\lambda} \leq \lambda \leq \overline{\lambda} < 1 \qquad \text{... (6)}$$

where $\underline{\lambda}$ and $\overline{\lambda}$ define real numbers, and $[\cdot]$ the integer part. Therefore, the shift cannot occur at the very beginning or the very end of the sample. The estimation of the structural shift is based on the regressions:

$$\vec{y}_t = \vec{v}_0 + \vec{v}_1 t + \vec{\delta} d_{t\tau} + \vec{A}_1 \vec{y}_{t-1} + \ldots + \vec{A}_1 \vec{y}_{t-p} + \varepsilon_{t\tau}, \quad t = p+1, \ldots, T \qquad \text{... (7)}$$

with $\vec{A}_i$, $i = 1, \ldots, p$ the $K \times K$ coefficient matrices, and ε_t the white noise K-dimensional error process. The estimator for the breakpoint is defined as:

$$\hat{\tau} = \arg\min_{\tau \in \mathrm{T}} \det\left(\sum_{t=p+1}^{T} \hat{\vec{\varepsilon}}_{t\tau} \hat{\vec{\varepsilon}}'_{t\tau} \right) \quad \text{... (8)}$$

with $\mathrm{T} = [T\underline{\lambda}, T\overline{\lambda}]$, and $\hat{\vec{\varepsilon}}_{t\tau}$ the least squares residuals of Eq. (7). Once the breakpoint $\hat{\tau}$ has been estimated, the data are adjusted as follows:

$$\hat{\vec{x}}_t = \vec{y}_t - \hat{\vec{\mu}}_0 - \hat{\vec{\mu}}_1 t + \hat{\vec{\delta}} d_{t\hat{\tau}} \quad \text{... (9)}$$

The test statistic writes:

$$LR(r) = T \sum_{j=r+1}^{N} \ln(1 + \hat{\lambda}_j) \quad \text{... (10)}$$

with corresponding critical values found in Trenkler (2003).

Estimation of the VECM

The error-correction model (ECM) writes:

$$\Delta X_t = \Pi_1 \Delta X_{t-1} + \ldots + \Pi_{p-1} \Delta X_{t-p+1} + \Pi_p X_{t-p} + \varepsilon_t \quad \text{... (11)}$$

where the matrices Π_i $(i = 1, \ldots, p)$ are of size $(N \times N)$. All variables are $I(0)$, except X_{t-p} which is $I(1)$. For all variables to be $I(0)$, $\Pi_p X_{t-p}$ needs to be $I(0)$ as well.

Let $\Pi_p = -\beta\alpha'$, where α' is an (r, N) matrix which contains r cointegration vectors, and β is an (N, r) matrix which contains the weights associated with each vector. If there exists r cointegration relationships, then $Rk(\Pi_p) = r$. Johansen's cointegration tests are based on this condition. We can thus rewrite Eq.(11):

$$\Delta X_t = \Pi_1 \Delta X_{t-1} + \ldots + \Pi_{p-1} \Delta X_{t-p+1} - \beta\alpha' X_{t-p} + \varepsilon_t \quad \text{... (12)}$$

The estimation of the corresponding vector error-correction model (VECM) is performed through maximum likelihood methods (Johansen and Juselius (1990), Johansen (1991)).

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