SHORT PAPER

Journal of Quantitative Economics, Vol. 11 Nos.1&2 (Combined), January-July 2013

MODELLING THE GROWTH PROCESS AS A VIRTUOUS CYCLE

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Abstract

The standard neo-classical model of economic growth entails a rate of income growth proportional to the income gapyet,in the most dramatic periods of economic transformation, a process of cumulative circular causation could be expected to lead to accelerating growth. Modelling the rate of change of income growth, instead of income growth itself, as proportional to the income gap gives testable predictions concerning the inverse cosine of the income gap. Estimates of real GDP per capita for six Western European nations,for which GDP data are available over the period 1830 to 1913, conform to these predictions.

Keywords: GDP, economic growth, cumulative circular causation

JEL Classification: E1, N1, O4

1. Introduction

Inherent in the standard neo-classical model of economic growth (Solow 1956), as well the more recent new growth theory adaptations of it (Romer 1986, 1990, Lucas 1988, Grossman and Helpman 1990, Mankiw, Romer and Weil 1992), is a rate of convergence to equilibrium proportional to the gap between a country's present position and its long-run equilibrium. It follows that, the poorer a country is, the more quickly it is supposed to be able to grow. This proposition does not sit well with the stylised facts of economic growth and such evidence as there is in support of it relies on the idea of growth clubs, groups of nations that share similar underlying capacities and access to technology (Baumol 1986, Durlauf and Johnson 1995, Quah 1997).

In thinking about those countries that have undergone important economic transitions, for example the currently developed nations which did so in the nineteenth century, it is intuitively appealing to regard the process as one involving a virtuous cycle in which initial gains are built on at an increasing rate. This idea has been expressed in the literature as cumulative circular causation (Kaldor 1972, Myrdal 1976, Young 1928); it implies that growth rates, rather than slowing with increasing income, will instead accelerate.

There are a number of crucial differences between these different ways of examining the process of economic growth. Central to the neo-classical view is the idea of equilibrium. In fact, all of mainstream growth theory, both old and new, is focused on the twin notions of existence and uniqueness of an equilibrium growth path. The evolutionary approach, in contrast, does not place equilibrium at its centre; on the contrary, criticism of this focus on equilibrium goes back

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over a century (Veblen 1898).As Castellaci (2007: 593) puts it, “Economic growth … is a never ending and ever changing process; it does not tend towards a steady state of balanced growth.”

Here we do not use the idea of equilibrium in the sense of neo-classical growth modelling, but we do draw on the notion of an income gap. In the neo-classical approach, the income gap refers to the gap between the steady state level of capital per workerand the current level of the same variable, or an equivalent gap in terms of income per capita.There is a problem in operationalizing this notion of equilibrium, for how can we know what the long-run state will be? In empirical work it is often simply assumed that some leader country represents the state to which others are catching up. Instead of measuring a gap with reference to a steady state level, we measure it from the level achieved at the end of some period of economic transformation. Such a level is not some theoretical value towards which an economy inevitably, although only asymptotically, converges, but rather is coincident with a point in time when some evolutionary process was interrupted by a major societal change such as an economic or financial crisis or war.

The outline of the remainder of this paper is as follows. In section 2 we set out the model, illustrating how its predictions differ from that of neo-classical growth theory. Section 3 tests these predictions against the experience of the six Western European nations for which reliable estimates of GDP per capita are available annually over the period 1830-1913, duringwhich they underwent an unprecedented period of economic growth ending on the outbreak of the Great War.

2. Model

In terms of the variable at the centre of modelling the growth process, income per capita, *y*, the idea of a virtuous cycle can be formalised as:

$$\ddot{y}=-\lambda g \text{ with } g=(y-y^{*})$$

wherey^{*}represents the value of *y*at the end of a period of economic transformation and λ is some positive constant.

This equation is well known in the physical sciences as representing simple harmonic motion and can be shown to be equivalent to:

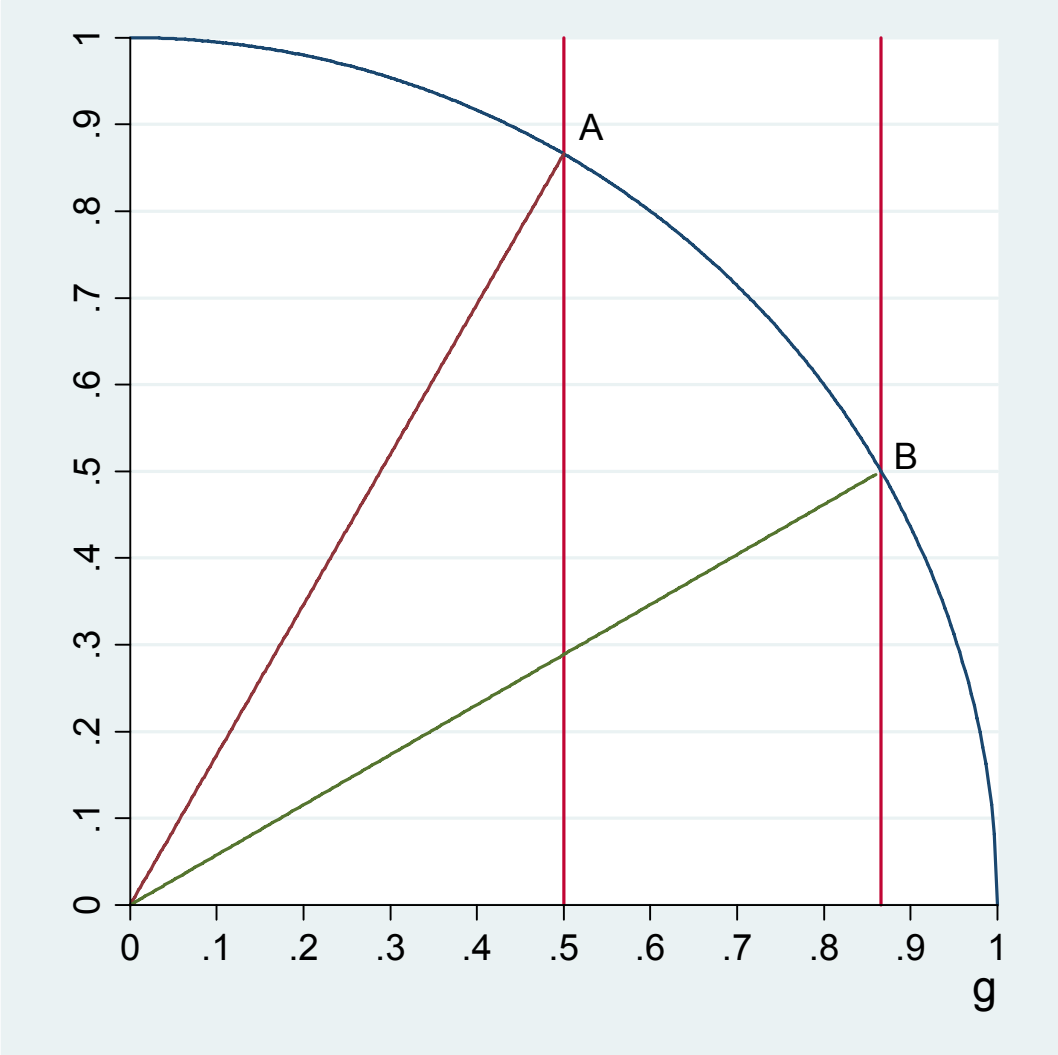
$$\dot{\theta}=\omega \text{ with } \lambda=\omega^{2} \text{and } g=\cos(\theta)$$

The connections amongst the variables are most easily seen by means of a diagram such as Figure1.

We measure the income gap, *g*, along the horizontal axis. At any given point in time, think of any given point on the unit circle, such as A, moving anti-clockwise at a constant rate, ω. The gap, found by projecting the point A onto the horizontal axis, closes more and more quickly as A continues to move at this constant rate.

As a way to illustrate the contrasting predictions of the standard neo-classical approach with this model, consider the example of two countries at different stages of development. Specifically, suppose that Country A is 50% of the way towards closing the gap with the leading country, while Country B still has 86.60% of the gap to close; equivalently, OA makes an angle of $\pi/3$ with the x-axis, while OB makes an angle of $\pi/6$. It is easy to see that the model predicts that A will have caught up with the leader at the same time that B has closed the gap to 50%. Put

another way, Country A doubles its income in the same time that Country B goes from 13.40% of the leader's income to 50% of the leader's income, representing an improvement of about 273% in its position. The improvement in B is greater in relative terms than the one in A, albeit off a lower base.A Solow-type model, since it embodies exponential decay, predicts instead that both countries approach the steady state asymptotically.



Note: OB is at $\pi/6$ and OA is at $\pi/3$ to the horizontal axis.

Figure 1. Points on the circle move at constant rate ω.

The most obvious implication of the model is that, in theory, $\ddot{\theta}=0$ since $\dot{\theta}=\omega$. In practice, if we had actual data, the second difference of θ would not be exactly zero but, if it were stationary

about a mean of zero, this would be consistent with a virtuous cycle in which economic growth accelerated, provided that θ itself trended upward over time, since $\theta=\omega t+\varepsilon$.

3. Data and testing

Maddison (2004) defines a number of epochs in economic history. He claims that the take-off point for the western developed nations is about 1820. Per capita GDP in the West grew at an average rate of 1.06% from 1820 to 1870 and 1.57% to 1913 before slowing considerably during the periods of the two World Wars and the intervening Great Depression. The period 1820 to 1913 therefore marks a period of transition for these economies; indeed, a period of remarkable and unprecedented economic change.

In the case of a small group of countries in Western Europe (the United Kingdom, Sweden, Norway, the Netherlands, France and Denmark), GDP data are available in Maddison (2010) going back annually to 1830 so that all of the countries have begun the transition process by then.Such data are ideal for testing the implications of the model.

Taking the income level of these countries in 1913as the basis for defining the income gap, we construct a measure of this gap for any given country, in terms of income per capita, in year *t*as: $gap_t = (y_{1913} - y_t)/y_{1913}$. We then define $\theta_t = \cos^{-1}(gap_t)$.

In terms of discrete data, wecan then test, for each country, whether:

1) The first difference of θ_t, $\Delta\theta_t$,is stationary.
2) The second difference of θ_t, $\Delta^2\theta_t$,is stationary about a mean of 0.
3) θ_t follows a deterministic trend; that is, the regression $\theta_t = \omega t + \varepsilon + u_t$ is a good fit.

If these conditions hold, we have evidence in favour of the proposition that, in these economies, from 1830 to 1913, growth of per capita income was accelerating up to the point of the outbreak of the Great War.

Table 1 presents the results of Phillips-Perron tests (Phillips and Perron 1988) of the null hypotheses of unit roots in $\Delta\theta_t$ and $\Delta^2\theta_t$ as well as a 95% confidence interval for the mean of $\Delta^2\theta_t$ and the R^2 of θ_t on a time trend.

The results are consistent for every country. The null of a unit root in either $\Delta\theta_t$ or $\Delta^2\theta_t$ is always rejected in favour of the alternative of stationarity, at better than the 1% level of significance. The second difference of θ_t is not significantly different from zero and between 93% and 99% of the variation in θ_t itself is explained by a simple time trend. We can conclude that these countries underwent an economic transformation characterised by a virtuous cycle in which economic growth built upon itself and accelerated.

4. Conclusion

The standard neo-classical approach to economic growth entails the prediction that poor countries will undergo economic transition by growing very quickly at first and then slowing down as they become richer. The evolutionary approach to growth would suggest, in contrast, that growth may start out slowly and build upon itselfin a virtuous cycle of circular cumulative

causation. Formalising this latter hypothesis as "change in growth is proportional to the gap" allows a simple test as to whether growth in any historical period did follow such a virtuous cycle. Applying such a test to the available data on the United Kingdom, Sweden, Norway, the Netherlands, France and Denmark, in the unprecedented modernisation period of 1830 to 1913, we find strong evidence that the GDP per capita of each of these countries accelerated.

Table 1.Tests of the virtuous cycle hypothesis

Country	$\Delta\theta$ *Phillips-Perron test stat*		$\Delta^2\theta$ *Phillips-Perron test stat*		*95% CI mean of* $\Delta^2\theta$	R^2 *of* θ *on year and constant*
	rho	*t*	*rho*	*t*		
United Kingdom	-62.413 (-19.476)	-7.512 (-3.535)	-87.913 (-19.458)	-14.472 (-3.537)	-0.0045 to 0.0049	0.9880
Sweden	-80.894 (-19.476)	-9.050 (-3.535)	-98.810 (-19.458)	-20.355 (-3.537)	-0.0057 to 0.0071	0.9308
Norway	-66.439 (-19.476)	-7.681 (-3.535)	-87.377 (-19.458)	-16.478 (-3.537)	-0.0037 to 0.0057	0.9732
Netherlands	-95.455 (-19.476)	-10.469 (-3.535)	-109.748 (-19.458)	-23.741 (-3.537)	-0.0062 to 0.0069	0.9530
France	-102.608 (-19.476)	-14.039 (-3.535)	114.340 (-19.458)	-26.830 (-3.537)	-0.0111 to 0.0107	0.9627
Denmark	-102.440 (-19.476)	-11.174 (-3.535)	-113.525 (-19.458)	-30.207 (-3.537)	-0.0048 to 0.0055	0.9308

Note: 1% critical values of Phillips-Perron tests in brackets

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