

HOW CAN LONG MEMORY IN VOLATILITY BE ELIMINATED IN PORTFOLIO OPTIMIZATION: AN EMPIRICAL EVIDENCE USING COPULAS

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Abstract

This paper focuses on the analysis of long-memory properties of copula-based time series. We empirically investigate the relation between copulas parameter modeling temporal dependence and dependence structure, using simulated and financial series. Our results prove the existence of a positive relation relying two Markov process X_{t+h} and Y_{t+h} to their dependence structure.

Keywords: Long memory process, copulas, measures of dependence, autocorrelations, persistence, volatility, GARCH.

JEL Classification: C22, C51, G10.

1. Introduction

Dependence is one of the most basic notions in different fields such as risk management and portfolios' selection. Various concepts of dependence are available in the literature (see, among others, the reviews in Joe (1997); Ango Nze (2004)). Cont (2001), and references therein, provide examples of time series and stochastic processes that exhibit dependence and autocorrelation properties ranging from short to long-memory.

According to Ibragimov et al. (2009a), we can present various definitions to dependence and long memory which differ in dependence measures used. Usually, long memory concept is based on the autocovariance or autocorrelation functions. The slow decay of these functions is a clear sign of the property of long-memory processes.³

Financial data are characterized by long memory in volatility process. This fact is commonly modeled using Fractionally Integrated Generalized ARCH models (FIGARCH). The FIGARCH(r, d, s) process has the infinite ARCH representation:

$$\sigma_t^2 = w(1 - \beta(L))^{-1} + \lambda(L)\varepsilon_t^2 \quad \dots (1)$$

where the polynomial $\lambda(L)$ is given by:

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³ See, among others, Lo (1991), Baillie et al. (1996), Hosking (1996).

$$\lambda(L) = \sum_{k=0}^{\infty} \lambda_k L^k = 1 - (1 - \beta(L))^{-1} \Phi(L) (1-L)^d \quad \dots (2)$$

FIGARCH(r,d,s) processes must meet some parameters restrictions to ensure positivity of the conditional variance σ_t^2 . In the case of a FIGARCH (1,d,1) process one must have $\beta_1 - d \leq \Phi_1 \leq \frac{2-d}{3}$; $d(\Phi_1 - \frac{1-d}{2}) \leq \beta_1(d + \alpha_1)$ and $\Phi_1 = \alpha_1 + \beta_1$.

However, the autocovariance or autocorrelation function can take into consideration only the linear dependence in the elliptical distributions and their existence is conditioned by having a data with finite second moments; which is not guaranteed for all financial data.⁴ Also, they are based on the classical correlation coefficient, which is a very poor statistical dependence measure that can only consider the linear dependence structure. Moreover, it is not the only possible dependence structure. Next, the value of the correlation coefficient of two return series differed between the value of the log of the same series, which cause a problem.⁵

This was a real problem until the study of Granger in 2003. His contribution consists of using the copulas theory to introduce a new definition to short memory and long memory processes. Copulas functions are a mathematical tool which goes back to the mid-1980s, but really introduced in finance and insurance industry by the pioneer paper of Paul Embrecht in the beginning of 2000. The formulation of these functions is done by Sklar (1959). Their importance is motivated by the evaluation of market risk, so that evaluating the dependence between risks can be done independently to marginals.

Let X_1, X_2 be continuous random variables with distribution function $G(x_1, x_2)$ and marginal distributions $G_1(x_1)$ and $G_2(x_2)$ correspondingly. For every $(x_1, x_2) \in [-\infty, +\infty]^2$ consider the point in $[0, 1]^3$ with coordinates $(G_1(x_1), G_2(x_2), G(x_1, x_2))$. This mapping from $[0, 1]^2$ to $[0, 1]$ is an 2-dimensional copula, or a bivariate copula. The following basic theorem (given in the bivariate case) is the main result in copula theory, e.g. Sklar (1959), and partially explains the importance of copulas, see also Nelsen (2006), p. 41.

Sklar's Theorem: *Let G be a bivariate dimensional distribution function with margins G_1, G_2 . Then there exists a 2-dimensional copula C such that for all $(x_1, x_2) \in [-\infty, +\infty]^2$*

$$G(x_1, x_2) = C(G_1(x_1), G_2(x_2)) \quad \dots (3)$$

Conversely, if C is a bivariate copula and G_1, G_2 are distribution functions, the function G defined by (3) is a 2-dimensional distribution function with margins G_1, G_2 . Furthermore, if the marginals are all continuous, C is unique. Otherwise, C is uniquely determined on $\text{Ran}G_1 \times \text{Ran}G_2$.

Firstly, we propose to model the dependence structure in bivariate case. Secondly, we attempt to model each series X_{t+h}, Y_{t+h} with various values of h lags (1 to 50) and model the

⁴ See, among others Embrechts (2002)

⁵ It is not invariant under strictly increasing linear transformations.

adequate copula. Iterating more than 100 times, we accumulate three vectors of copula parameters. Finally, we look for an explicit function by which we can link temporal dependence to dependence structure.

Our paper is the continuity of the Ibragimov et al. (2009b)'s research.⁶ It has many similarities and differences. The fundamental similarity is in the technical aspect that can allow making comparison. Many differences are introduced (1) to focus on the volatility process rather than the mean process (2) to analyze the dependence structure (3) to adopt the technique of McNeil, Frey and Embrechts in 2005 by fitting the copula at each h lag rather than the discretization method based on different grid points (4) to find the link between copulas parameter modeling dependence structure and temporal dependence. The remainder of this paper is as follow. Section 2 reports the main results of Granger (2003). Section 3 focuses on empirical study using both simulated and financial indexes and gives example based on portfolio optimization. Section 4 concludes.

2. Copula Based Approach to Long Memory: Review of Main Definitions and Properties

Granger (2003) proposes definitions of long memory and short memory processes $\{X_t\}_{-\infty}^{+\infty}$ using the (copula-linked) Hellinger measure of dependence $H(t,h) = H_{X_t, X_{t+h}}$ between the r.v.'s X_t and X_{t+h} . He suggests calling a process X_t long or short memory depending on whether,⁷ for some constant $A > 0$:

$$H(t,h) \sim Ah^{-p}, h \rightarrow \infty \quad \dots (4)$$

where $p > 0$, or $H(t,h) = O(\exp(-Ah)), h \rightarrow \infty$.

Granger (2003) further indicates that the difficulty with the above definition is that it depends on a particular measure of dependence and it has to be shown that some general rule applies. He also indicates the possibility of using other measures of dependence and remarks that there seems to be no single measure that provides a dominant alternative to autocovariances and autocorrelations, while the measure H has the advantage of simplicity and of having the link to copulas.

Here the Hellinger distance is used as the autocovariance function which proves an hyperbolic decay as $h \rightarrow \infty$. According to one of the commonly employed definitions, a weakly stationary process $\{X_t\}_{-\infty}^{+\infty}$ with autocovariance function $\gamma(h) = \text{cov}(X_t, X_{t+h})$ is said to have long

memory if $\sum_{h=-\infty}^{+\infty} |\gamma(h)|$ is divergent, and to have short memory otherwise.

⁶ It is based on the discussion paper number 2160 of Rustam Ibragimov and George Lentzas published by Harvard Institute of Economic Research in August 2008.

⁷ The reader can refer to Ibragimov et al.(2009) for more details.

The process $\{X_t\}_{-\infty}^{+\infty}$ with long range dependence is both stationary and invertible if $|d| < 0.5$. For $0 < d < 0.5$, the process exhibits long memory. For $-0.5 < d < 0$ the process is said to exhibit intermediate memory (anti-persistence) or long range negative dependence.

More precisely, a weakly stationary process $\{X_t\}_{-\infty}^{+\infty}$ is said to exhibit long memory or long range dependence if:

$$\gamma(h) \sim \begin{cases} h^\beta l(h) & \text{for } \beta \in (-1, 0) \\ -h^\beta l(h) & \text{for } \beta \in (-2, -1) \end{cases} \quad \dots (5)$$

As $h \rightarrow \infty$.

where $l(h)$ is a slowly varying function at infinity: $\frac{l(\lambda h)}{l(h)} \rightarrow 1$ as $h \rightarrow \infty$ for all $\lambda > 0$. Then we have

$$\beta = 2d - 1 \text{ and } l(h) \equiv A.$$

The decay of autocovariances is qualitatively different from and is significantly slower than that of autocovariances of stationary and invertible ARMA process. Autocovariance functions $\gamma(h)$ of such process satisfy:

$$\gamma(h) = O(\exp(-Ah)) \quad \dots (6)$$

As $h \rightarrow \infty$ for $A > 0$.

According to our reference work, a number of divergence measures for random variables can be expressed or defined using their copulas such as Pearson's coefficient ($\rho_{X,Y}$), the relative entropy (or Kullback-Leiber mutual information) ($\bar{\delta}_{X,Y}$) and Hellinger measure of dependence ($H_{X,Y}$).

$$D_{X,Y}^\psi = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi \left(\frac{f_{X,Y}(x,y)}{f_X(x)f_Y(y)} \right) f_X(x)f_Y(y) dx dy \quad \dots (7)$$

where ψ is a strictly convex function on \Re satisfying $\psi(1) = 0$. The multivariate Pearson's coefficient correspond to $\psi(x) = x^2 - 1$ and the relative entropy is obtained with $\psi(x) = x \ln(x)$. The choice $\psi(x) = \frac{1}{2} \left(1 - x^{1/2} \right)^2$ leads to Hellinger measure of dependence considered in Granger (2003) and Granger (2004). As discussed in Granger (2004), Ibragimov and Lentzas affirm that the Hellinger measure $H_{X,Y}$ and its scaled versions are rather unique among divergence measures since they satisfy the triangular inequality and are proper measures of distance.

$$H_{X,Y} = \left(\frac{1}{2} \right) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(f_{X,Y}^{1/2}(x,y) - f_X^{1/2}(x)f_Y^{1/2}(y) \right)^2 dx dy \quad \dots (8)$$

$$H_{X,Y} = 1 - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(f_{X,Y}^{1/2}(x,y) f_X^{1/2}(x) f_Y^{1/2}(y) \right) dx dy \quad \dots (9)$$

Let $C(u, v) = C_{X,Y}(u, v)$ be the copula of X, Y and U, V denotes iid random variables uniformly distributed. We focus on the Hellinger distance which can be written in terms of the copula density $c(u, v) = \partial^2 C(u, v) / \partial u \partial v$ as follows:

$$H_{X,Y} = H(C) = \frac{1}{2} \int_0^1 \int_0^1 (c^{1/2}(u, v) - 1)^2 \, dudv = \frac{1}{2} E(c^{1/2}(U, V) - 1)^2 \quad \dots (10)$$

$$H_{X,Y} = H(C) = 1 - E c^{1/2}(U, V) \quad \dots (11)$$

The measure, generally used, is the hyperbolic decay using $\kappa(h)$ which represents the deviation from independence:

$$\kappa(h) = \kappa(C) = \int_0^1 \int_0^1 |C(u, v) - uv| \, dudv = E |C(U, V) - UV| \quad \dots (12)$$

The next section completes justification of definitions of long memory discussed above and we choose to use $\kappa(h) = \kappa_{X_t, X_{t+h}}$ as the measure of dependence, which is more intuitive than divergence measure.

3. Clayton Copula-based Long Memory Processes: An Empirical Validation

3.1 Simulation Study

We start with many stationary FIGARCH $(0, d, 0)$ for $d \in [0.1, 0.42]$ and select two discrete series, namely X_t and Y_t (vectors of 1000 observations). Each series will be decomposed into 50 series X_{t-h} when h varies from 0 to 50. Thus, we can show long memory aspect as function of h lag. Next, we concentrate on estimating the adequate copulas between X_t and X_{t-h} , Y_t and Y_{t-h} then finally between X_{t-h} and Y_{t-h} . The estimated parameters are in figure 1.

Next, we deal with the Clayton copulas $C_{t,t+1}(u, v)$ and we then calculate the implied copulas $C_{t,t+h}(u, v)$ when $h = 1, \dots, 50$ and report a number of different values of Clayton copula parameters.

We numerically evaluate the distance $\kappa(h) = \kappa_{X_t, X_{t+h}}$ and report results. Overall, the distance $\kappa(h)$ decreases slowly and uniformly as the time lag h increases, showing clear evidence of κ -long memory.⁸ Table 1 summarizes the estimated parameters and the values in parentheses are t-values.

⁸ Since we are approximating each copula, error is inevitably introduced in our estimates.

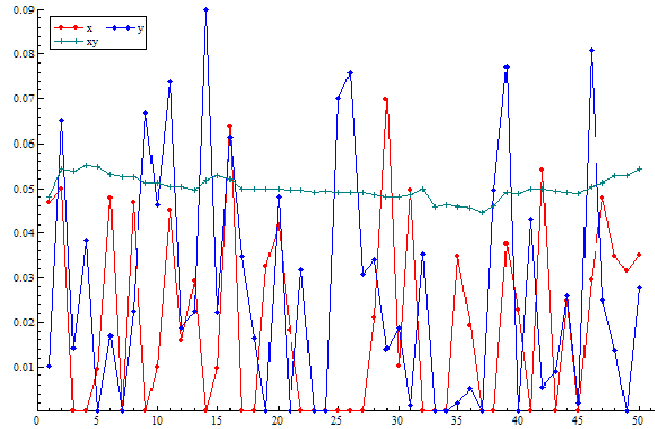


Figure 1. The estimated Clayton copula parameters modeling both long memory and dependence structure.

Table 1. ARMA(p, q) - FIGARCH(u, d, v) estimations.

	Cst (M)	AR	MA	Cst(V)	ARCH(1)	GARCH(1)	d - FIGARCH
X	0	0	0	0.110909(4.106)	0	0	0.299226(4.803)
Y	0	0	0	0.220985(2.325)	0	0	0.424813(6.842)

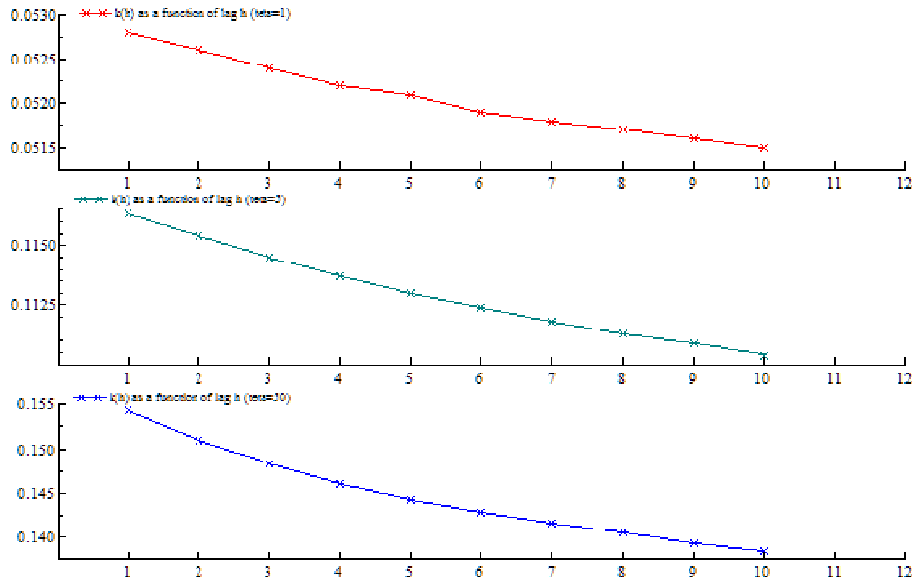


Figure 2. The measure $\kappa(h)$ as function of lag h with $\theta = 1, 5, 10$

To assess the value of the long memory parameter p in the characteristic long memory relation:

$$\kappa(h) \sim Ah^{-p} \quad \dots (13)$$

We regress the logarithm of the left hand side of the initially mentioned equation on its right hand side for $\theta = 1, 5, 50$ respectively. The results are statistically significant using Ordinary Least Squared technique.

For X_t simulated vector:

$$\text{Ln}(\kappa(h)) = -3.19 - 0.014 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -2.56 - 0.0195 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -2.43 - 0.023 \text{Ln}(h)$$

We can conclude directly that the higher the θ , higher is the long memory parameter, 0.014, 0.0195 and the last 0.023. Thus, we have a positive relation between long memory parameter and Clayton copula parameter modeling long memory.

The same remarks with the second simulated vector Y_t :

$$\text{Ln}(\kappa(h)) = -3.17 - 0.0196 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -2.54 - 0.0268 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -2.42 - 0.02819 \text{Ln}(h)$$

Long memory parameters values of p indicate very slow hyperbolic decay of the distance $\kappa(h)$ to zero with h . Our results suggest that the higher the Clayton parameter θ , higher the long memory parameter is. It is the first issue dealing with more detailed analysis between long memory and copulas. Our finding seems to be logical so that when copula parameter goes to -1, the copula tends to be independence copula and $\kappa(h)$ tend to 0. Thus, to compute the estimated long memory process from FIGARCH model, we must have a bigger θ parameter.⁹

3.2 Clayton Copula Modeling Dependence Structure End Temporal Dependence

Now, we concentrate on the relation between vector of Clayton copula parameter (modeling dependence structure between X_{t-h} , Y_{t-h}) and the two vectors of Clayton copula parameter (modeling long memory effect for both X_{t-h} and Y_{t-h}). Also, to the best of our knowledge, we are the first to develop this idea and try to influence on long memory parameter using dependence structure parameter. Mathematically, we have two cases: 1) Clayton copulas modeling both dependence structure and temporal dependence have the same parameter function 2) it is different.

⁹ These results are showed in different FIGARCH (0,d,0) process.

3.2.1 The function of Clayton copula parameters are the same

Let U_t, V_t, W_t and Z_t the transformed uniformly data of respectively X_t, X_{t-h}, Y_t and Y_{t-h} in $[0, 1]$:

$$C(F^{-1}(X_t), F^{-1}(X_{t-h})) = (U_t^{-\theta} + V_t^{-\theta} - 1)^{-1/\theta}$$

$$C(F^{-1}(Y_t), F^{-1}(Y_{t-h})) = (W_t^{-\theta} + Z_t^{-\theta} - 1)^{-1/\theta}$$

$$C(F^{-1}(X_t), F^{-1}(Y_t)) = (U_t^{-\theta} + W_t^{-\theta} - 1)^{-1/\theta}$$

Then we have:

$$\begin{aligned} & C(F^{-1}(X_t), F^{-1}(Y_t)) \\ &= ((U_t^{-\theta} + V_t^{-\theta} - 1) + (W_t^{-\theta} + Z_t^{-\theta} - 1) - (V_t^{-\theta} + Z_t^{-\theta} - 1))^{-1/\theta} \\ &= (C(F^{-1}(X_t), F^{-1}(X_{t-h})) + C(F^{-1}(Y_t), F^{-1}(Y_{t-h})) - C(F^{-1}(X_{t-h}), F^{-1}(Y_{t-h})))^{-1/\theta} \end{aligned}$$

And finally:

$$C(F^{-1}(X_t), F^{-1}(Y_t))^{-\theta} + C(F^{-1}(X_{t-h}), F^{-1}(Y_{t-h}))^{-\theta} = C(F^{-1}(X_t), F^{-1}(X_{t-h}))^{-\theta} + C(F^{-1}(Y_t), F^{-1}(Y_{t-h}))^{-\theta}$$

The first part of this equality represents Clayton copula function modeling dependence structure between X_t, Y_t and X_{t-h}, Y_{t-h} . While the second represents Clayton copula function modeling long memory property related to X_t, X_{t-h} and Y_t, Y_{t-h} . Thus, long memory parameter (which has a positive relation with Clayton copula parameter modeling long memory process) has a clear relation to Clayton copula parameter modeling dependence structure between long memory series. Thus, each of these parameters can influence the second: if copula parameter increases the two parts of the following equality will decrease and vice versa.

3.2.2 The function of Clayton copula parameters are different

If we have different Clayton copula parameter functions for long memory process as well as for dependence structure, then we must turn to focus on what relation can exist between these three vectors of parameters. Many ideas can be tested such as linear dependence structure, OLS regression and non linear dependence structure. The linear hypothesis dependence structure seems to be very restrictive: for our arbitrary simulated data we have a very small correlation coefficient between two vectors of parameter of "long memory" near to 0.0289 and 0.1549 and 0.2138 between each vector of "long memory" and that of the "dependence structure". Then, the OLS regression' results show a non significant estimators for all tested regression models (We show the different possible case of the endogenous variable and the regressors). Thus, it could not be the adequate method. Next, we turn to estimate eventually non linear dependence between these three vectors of Clayton copula parameters. Our results suggest that there exists a positive nonlinear dependence structure, modeled via Gumbel copula with stronger parameter value 1023, between vector of copula parameter of long range dependence and this dependence structure.

From these results, we can conclude that we have a strong positive relation between each pair of our study. Each pair of vectors is modeled using the Gumbel Copula showing a positive dependence structure with a higher parameter. Thus, we have a strong positive relation between Clayton copula modeling dependence structure and Clayton copula modeling long

memory process. Consequently, we have a positive relation between long memory parameter and Clayton copula parameter modeling dependence structure between these series. Thus, we can influence the unobserved long memory parameter using the observed dependence structure. Thus, if we wish to reduce long memory parameter we must reduce copula parameter of dependence structure. In our case, when Clayton copula parameter goes to zero the Clayton copula tends to the independence copula.

3.3 Volatility Modeling Using Copulas: Application of Portfolio Optimization

Let us first recall the basic concept introduced in portfolio management. Then, we expose the Principal Component Analysis and study its performance on the optimization program.

3.3.1 Mean-Variance efficient portfolio

Modern portfolio theory is a theory of investment which attempts to minimize risk for a given level of expected return. Thus, it is called mean-variance analysis. Markowitz's analysis explains how to find the best possible diversification strategy under certain assumptions and for specific quantitative definitions of risk and return. His contribution, the mean-variance analysis, is a mathematical formulation of the concept of diversification in investing. We can formulate the program as follows:

$$\begin{cases} \text{Min } \sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_{ij} \\ \text{s/c} \\ \sum_{i=1}^N X_i E(R_i) = E(R_p) \\ \sum_{i=1}^N X_i = 1 \end{cases}$$

where X_i and X_j denote the amounts invested in different assets $i, j = 1, \dots, N$, R_i denote the return of the assets, R_p denote the desired portfolio return and σ_p^2 denote the variance covariance matrix.

The solution of the associated optimization problem bears a convenient graphical representation in the standard deviation and expected return plan. Once each asset return is identified with a point in this plan, the returns of the variance minimizing portfolios span a characteristic hyperbolic curve, called the mean-variance frontier of asset returns. Portfolios whose returns are on the mean-variance frontier are called efficient.

3.3.2 Principal Component Analysis

Principal Component Analysis is a statistical technique that allows us to derive one or more summary measures ("principal components") from a set of real indicators. Each principal component is a weighted average of the underlying indicators. We use some mathematics tool to choose weights such that the principal component accounts for a maximum amount of the

variance in the underlying indicators. Principal Component Analysis basically relies on some matrix algebra. We have p variables observed across i cross-sectional units. Their correlation matrix is:

$$\Sigma_{p \times p} = \begin{pmatrix} 1 & \sigma_{12} & \dots \\ \sigma_{21} & \ddots & \vdots \\ \vdots & \dots & 1 \end{pmatrix}$$

Then we can write:

$$\Sigma_{p \times p} = C \Lambda C' = \begin{pmatrix} c_{11} & \dots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{p1} & \dots & c_{pp} \end{pmatrix} \begin{pmatrix} \lambda_{11} & 0 & \dots \\ 0 & \ddots & \vdots \\ \vdots & \dots & \lambda_{pp} \end{pmatrix} \begin{pmatrix} c_{11} & \dots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{p1} & \dots & c_{pp} \end{pmatrix}$$

Next, we turn to the p indicators and we have:

$$\Sigma_{p \times p} \approx \Sigma_{p \times p}^* = P P'$$

When P is matrix of n columns from which we can construct n principal component. Each element of P is the proportion of the variance of each original indicator that a particular principal component accounts for. Next, we can produce the scores. What is important is that multiple principal components are uncorrelated.

3.3.3 Efficient portfolio using financial data with long memory:

Our data are collected from different international financial market: developed financial market,¹⁰ emerging market¹¹ and under developed market.¹² Our source is Bank of Canada and the period is from 03-01-2005 to 31-12-2012. Our choice is arbitrary and it is for comparison purposes.¹³

Table 2. Statistics of the data

	<i>DLJPY</i>	<i>DLRUB</i>	<i>DLTND</i>
Mean	0.0116E-03	0.152E-03	0.2245E-03
Standard deviation	0.0102	0.0066	0.0062
Maximum	0.0696	0.0522	0.0359
Minimum	-0.0635	-0.0364	-0.0368
Skewness	-0.2244	0.6020	-0.1156
Kurtosis	8.4086	8.8871	5.6326

First, we attempt to test the asymptotic decline of $\kappa(h)$ for the Japanese exchange rate. Second, we attempt to construct an efficient Markowitz's frontier using CVaR-optimization program and third we will conclude.

¹⁰ represented by the Japanese exchange rate reported to the Canadian currency (JPY/CAD)

¹¹ represented by the Russian exchange rate reported to the Canadian currency (RUB/CAD)

¹² represented by the Tunisian exchange rate reported to the Canadian currency (TND/CAD)

¹³ The normality hypothesis is rejected for all series using the Jbtest (P is less than the smallest tabulated value) already implemented in MATLAB R2008a.

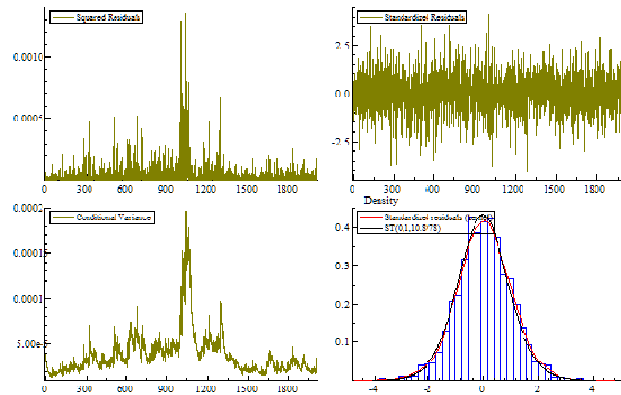


Figure 3. Estimation' results of the Tunisian index

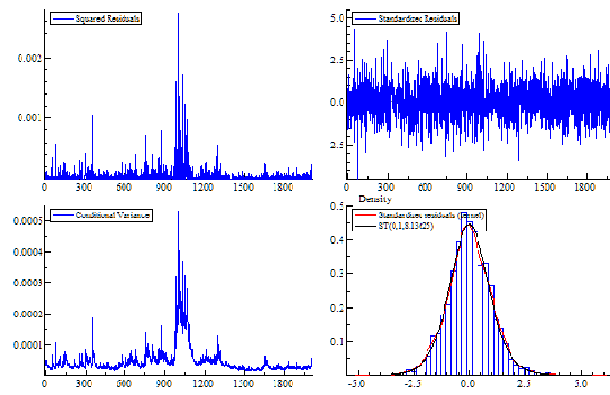


Figure 4. Estimation 'results of Russian index.

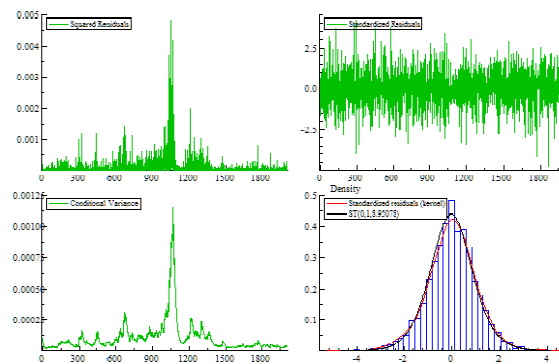


Figure 5. Estimation' results of the Japanese index.

Results show a clear decline of $\kappa(h)$ with lag h and this phenomenon is readable as higher as θ is. We regress the Log of the endogenous variable $\kappa(h)$ to the regressor Log of lag h to compute an approximation of long memory parameter. Results of long memory' estimations are represented in Table 4.¹⁴

Table 3. Estimation results of financial indexes.

	<i>DLTND</i>	<i>DLRUB</i>	<i>DLJPY</i>
Cst(M)	0	0	0
AR(1)	0	0	0.92085(15.34)
MA(1)	0	0	-0.931069(-17.58)
Cst(V)	0.286877(3.412)	0.422651(2.464)	0
d-FIGARCH	0.470337(3.351)	0.469496(4.252)	0.794244(13.46)
ARCH(Phi1)	0.406954(4.17)	0.441789(4.73)	0.141036(2.59)
GARCH(Beta1)	0.803666(11.76)	0.759576(11.09)	0.883499(40.35)

Considering Japanese exchange rate for $\theta = 1, 5, 50$, then we have:

$$\text{Ln}(\kappa(h)) = -2.93842 - 0.0112384 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -2.14549 - 0.0233559 \text{Ln}(h)$$

$$\text{Ln}(\kappa(h)) = -1.86094 - 0.0482857 \text{Ln}(h)$$

As the simulated study, our empirical financial study proves that long memory parameter increase when copula parameter increases. Next, we apply results and compute two orthogonal series from the initial raw series and apply the CVaR-optimization program.

For a comparison purposes, we plot all risk measures as function of portfolio numbers. Results show that the risk level in the two principal components is closer to the residual series rather than the stationary data. Thus, with the mathematic transformation, we can produce an uncorrelated data simply using the ACP method and then reduce shocks' effects on financial data. Thus, when investor composes his/her portfolio based on principal components,¹⁵ he/she can observe the real risk, which is higher for the stationary portfolio. The use of the principal components to construct the efficient frontier can outperform the classical analysis.

¹⁴ The value in parenthesis is the t-value. We should note that all series are estimated with a student distribution of residuals.

¹⁵ We have the same case using the normalized Principal Component Analysis.

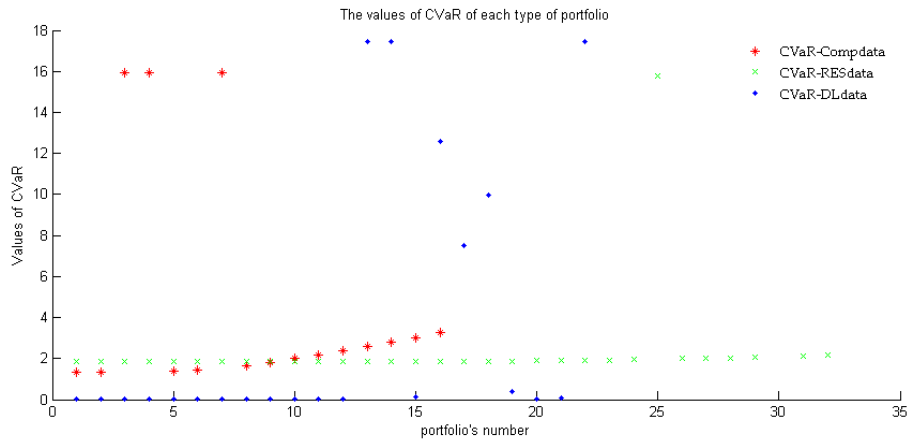


Figure 6. The values of CVaR for the stationary, residuals and principal components of financial indexes.

4. Conclusion

In this paper, we empirically investigate the relation between long memory and copulas and we checked if this latter relation has an impact on optimization portfolios. For that, we conduct our analysis in two steps. In a first step, we examine separately long memory effect and dependence structure using copulas theory. Our results, using both simulated and financial data, show evidence of a positive relation relying X_t and X_{t-h} presenting long range dependence in volatility process to their dependence structure.

In a second step, we study how results of first step can affect portfolio optimization. We consider the classical Mean-Variance analysis and draw the efficient frontier using different risk measures. Then, we consider new indicators (using principal component), constructed from our initial financial data, and draw a new frontier. The empirical results show that the risk level using the Principal Component Analysis is closer to the residual measures; which implies the reduction of long memory effect. In spite of the obtained results, we conclude that the relation between long memory and copulas affects risk measures and then investors' preferences.

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