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CONTEMPORARY RELEVANCE AND ONGOING CONTROVERSIES RELATED TO THE CES PRODUCTION FUNCTION

T.V.S.RAMAMOHAN RAO[1]

"There are very few instances where empirical or theoretical economic considerations have been used to determine functional form or shape restrictions beyond the general requirements of monotonicity and concavity (or convexity). Rather, attention has focused on building flexibility into functional forms in the hope that the 'true' form will be close to some element of the class estimated. Perhaps it is the best that can be done, but surely it would be more satisfying if functional form restrictions could be justified on the basis of experimental or empirical evidence."

– Starett (1987, p.93).

Abstract

The CES specification has been an important watershed in the development of the production function and its applications over the past fifty years. While the original formulation and estimation were rooted in sound economic theory many issues have been unresolved. In particular, the economic sources of the elasticity of substitution and the meaning of the "returns to scale" parameter remained ambiguous. Similarly, the efficient method of estimating the elasticity of substitution is as yet not clear. The primary reason is that several proposed methods of estimating the non-linear CES function did not pay attention to the economic decision processes underlying the observed choices from which

[1] Emeritus Professor, Indian Institute of Technology, Kanpur. Email: mrao@iitk.ac.in

I appreciate the encouragement from Dilip Nachane, P.K.Chaubey and Bandi Kamaiah. I wish to dedicate this note to my teacher Prof.B.S.Minhas as a token acknowledgement of his genius. Few other economists from India have been able to reach such heights. On a personal front three events are still etched in my mind. First, within a few minutes after I completed my presentation of the M.Stat thesis Prof.Minhas sent word for me. I was very scared since I could not answer many of the questions that he and Prof.T.N.Srinivasan posed. I was quite surprised when he gave me a letter of appointment to join them at the ISI, New Delhi. Second, he was always uncompromising as a professional economist. In my personal experience I will recall his stinging comment that the potential he found in me while I was a young student was never realized. I accepted it with all humility. Third, just before his death I had the good fortune of traveling with him from Bangalore to New Delhi. He invited me to his home that evening when he realized that I had four hours before I could catch a train to Kanpur. We had the most demanding academic discussion I can recall. Very few other professionals have been so generous to their students and associates. All of us who got to know him over the years miss him sorely.

estimation begins. Despite these shortcomings substitution and its applications have been so important that a whole lot of applications have been conceptualized. Several important insights have been recorded from the empirical work. However, it was found that the elasticity of substitution concept must be supplemented, or supplanted altogether, by several other explanations, equally rooted in economic theory and well documented empirical regularities, to provide a complete understanding of empirical phenomena. We may need many more years to finally decide which of the developments are worthwhile and which ones deserve discarding. It may take another genius of the stature of Minhas to steer the research out its current turbulence.

1. Introduction

2011 marks the 50^{th} year of publication of the Arrow et al (1961) paper where the CES production function was introduced. The hallmark of the development of the CES production function was that it was grounded in empirical observation, has a sound basis in economic theory, and was found necessary in explaining several economic phenomena. To the extent I am aware of it no other paper was considered relevant in such a large number of applications and cited so regularly over such an extended time horizon. However, despite its practical and empirical importance it has been difficult to resolve many issues related to the CES function. The present study attempts to put in perspective a few issues that have been highlighted between 2001 and 2011.[2] Even this is a formidable task if a comprehensive presentation is attempted. Hence, I will highlight only some issues that I am familiar with.

There are several observable trends in the development of the concept of elasticity of substitution. First, every stage of the development was in response to certain empirically observed facts that previous versions of the CES functions did not incorporate in their specification. The attempt was invariably to begin analysis with assumptions that were grounded in empirical reality. Second, there has been recognition that empirical reality that needs to be incorporated reveals itself only to the extent we look for it after the need is recognized. Third, we tend to look for new regularities only when the postulated functions and their implications do not correspond to certain stylized facts that we are trying to explain. Fourth, per force, there is an acknowledgement that the elasticity of substitution per se cannot offer a complete and convincing answer to every issue we are analyzing.[3] The initial attempt will be to augment the CES related models and make efforts to improve the explanation. However, as a natural logical process, it is acknowledged that superior explanations, based on premises other than CES, are available in specific contexts. The replacement is affected even if belatedly. However, it is interesting to note that many research workers are unwilling to give up their old habits once they developed certain vested interest. Perhaps this retarded the progress and seeking the right directions of change. I will make a sincere attempt to highlight these developments of the CES function.

2 I will invoke previous literature only when it is essential. So far as I know only a few survey articles have appeared on specific issues and applications. One such is Chang (2011).

3 Indeed these were the main themes in Minhas' presidential address to the TIES in 1993. It was reprinted in Minhas (2008).

The rest of the note is organized as follows. In section 2 we will highlight some of the major developments in the specification of the CES function as they relate to the empirical realism of assumptions and their capacity to explain other empirically observed facts. Section 3 will offer a brief outline of the problems of estimation. The major purpose is to highlight the stark contrast between methods that keep a close watch on the economic processes that generated the observed data and purely statistical and numerical considerations leading to econometric practices. It is surprising that we do not as yet have a clear definition of methods of estimating the elasticity of substitution as applicable to different problem structures. Section 4 highlights a few relatively recent developments in the application of CES functions. Our basic purpose is to identify the trends alluded to in the previous paragraph. Section 5 highlights the achievements so far and suggests a few lines of investigation that are likely to make the CES a more potent tool in econometric analysis.

2. Specification

Concerns regarding diminishing returns to factor use and returns to scale dominated the early investigations about the properties of the production function. This is evident from Gordon (2011), Humphrey (1997) and Mishra (2007). In particular, the major argument in the specification of the Cobb-Douglas production function was that

$$y/x_1 = A\,(x_2/x_1)^a\ ;\ 0 \le a \le 1$$

where y = output, and x_1,x_2 are two factors of production. Essentially it was based on the empirical observation that the intensive use of x_2, i.e., an increase in x_2/x_1, created diminishing returns to the productivity of x_1. There was no explicit attention to the elasticity of substitution or its implications for economic choice.

Historically a production function is a technological relationship between output and inputs. As such economic choice of inputs is not the source of substitution. However, when Hicks (1932) initiated an analysis of the curvature of isoquants and how factor incomes vary with capital accumulation he defined the elasticity of substitution as if it is a result of such choice. This practice, though confusing, has been perpetuated. However, prior to the specification of the CES production function, functions with zero or unitary elasticity of substitution dominated.

The major departure in Minhas (1963) and Arrow et al (1961) was the assertion that capital labor substitution was ubiquitous in most practical contexts. Minhas (1963) argued that it was, however, not a constant across different firms or activities. Instead, it varies from one firm to another even if it is a constant for any one of them. The CES production function acknowledged this variety and the necessity to capture it in a unified specification. The Arrow et al (1961) specification

$$y = (ax_1^{\alpha} + bx_2^{\alpha})^{1/\alpha}$$

implied the elasticity of substitution $\sigma = 1/(1-\alpha)$; $-\infty < \alpha < 1$. This function also allowed the possibility that $y > 0$ even when $x_2 = 0$. Clearly, even this was an advantage over the Cobb-Douglas production function.

Multiproduct specifications, exemplified by Baumol et al (1982), postulated that

$$r = \text{revenue of the firm} = (ay_1^{\alpha} + by_2^{\alpha})^{\gamma/\alpha}$$

for given x_1,x_2. In this specification the emphasis shifted to the substitution between y_1 and y_2 and a specification of economies of scope (since scale changes in x_1 and x_2 were not explicit).

Three aspects about the CES specification must be highlighted. First, it is generally suggested that a and b are factor shares and that a+b = 1 when constant returns to scale obtain. This is an invalid argument. For,

s_1 = payment to x_1 based on its marginal product

$= ax_1^{\alpha} (ax_1^{\alpha} + bx_2^{\alpha})^{1/\alpha - 1}$

$= ax_1^{\alpha}y/ (ax_1^{\alpha} + bx_2^{\alpha})$

Hence, $s_1/y \neq a$. Also note that $s_1 + s_2 = y$ irrespective of whether a+b = 1 or not.[4] Second, interpreting γ as returns to scale may not be the best because its effect on y is evident even when x_1 and x_2 do not vary proportionately. Arrow et al (1961, p.247) acknowledged this limitation. For, as they put it, “another loose end (of their analysis) has to do with the question of returns to scale.” If, ex post, x_1 and x_2 vary proportionately then γ may capture returns to scale. Otherwise it represents synergies and/or economies of scope. Rao (2011a), for instance, interpreted γ to reflect synergies due to organizational coordination. An apriori judgement is not warranted. Third, there is irrefutable empirical evidence that both substitutable and complementary products can offer a market advantage to a firm. r will increase for value of $\sigma < 1$ as well as $\sigma > 1$. In fact, as was demonstrated in Rao (2011b), $\sigma = 1$ corresponds to the case where r is minimum. Given this observation it is surprising that the Cobb-Douglas production function has been so consistently successful in a large amount of empirical work.

Subsequent extensions, including those related to variable elasticity of substitution, were too mechanical and devoid of economic rationale. This observation applies to the Diewert (1971) function, the translog function of Christensen et al (1973) and so on as well.

The issue of ex ante technical substitution vs. the elasticity of substitution indicated by ex post choice was always behind the scene since it has very important economic implications. Two opposing arguments are discernible. First, while the firm can choose the appropriate technology depending on factor prices ex ante, there will be no substitutability between x_1 and x_2 ex post because the chosen technology corresponds to a fixed x_1/x_2 ratio. See, for example, Nakamura (2009, p.465). Second, quite the opposite may be suggested. All techniques are rigidly fixed coefficient ex ante. However, as p_1 increases firms use those techniques that use more x_2 ex post. This type of argument is explicit in Robinson (1965). A clear specification of the distinction between ex ante and ex post definition of the elasticity of substitution is not available in the current literature despite this acknowledgement.

The following approach appears promising. Consider the CES function

$y = (ax_1^{\alpha} + bx_2^{\alpha})^{1/\alpha}$

By the logic of technical substitution the firm will substitute x_1 for x_2 if the contribution of x_1 to output increases in greater proportion to the contribution of x_2. The possibility of ex ante

[4] In fact, while deriving their equation (12) Arrow et al (1961, p. 230) noted that this is merely a normalization rule for algebraic convenience. Thus, their factor shares interpretation is an aberration. Since it is valid in the context of a Cobb-Douglas production function the temptation to accept the same in the context of CES was very significant. Its persistence demonstrates how uncritically the economics profession accepts certain powerful statements.

complementarity of x_1,x_2 suggests that such substitution may well be negative. Consider the contribution c_1 of x_1 to y to formalize this definition of ex ante substitution. It may be represented by

$$c_1 = ax_1^{\alpha} (ax_1^{\alpha} + bx_2^{\alpha})^{1/\alpha - 1}$$

since each unit of x_1 contributes its marginal product to y. Define c_2 analogously. It follows that

$$c_1/c_2 = (a/b)\ (x_1/x_2)^{\alpha}$$

An ex ante measure of the elasticity of substitution, purely on technical grounds and without invoking any economic choice, can be specified as

$$\sigma^* = d \log (c_1/c_2)/ d \log (x_1/x_2) = \alpha$$

The production function remains a CES even with this ex ante specification of the elasticity of substitution.

The economic content underlying σ^* may now be elucidated. Suppose x_1 and x_2 are strictly complementary. Then, λx_1, λx_2 of inputs will yield an output λy and $[c_1(\lambda)/c_2(\lambda)]$ does not contain any λ. In other words, $\sigma^* = 0$ represents strict complementarity. Suppose, instead, that x_1,x_2 are substitutable. Then, as x_1 increases the x_2 utilized along an isoquant will decrease. The corresponding increase in c_1 and reduction in c_2 make c_1/c_2 larger. Hence, $\sigma^* = \alpha > 0$. In general, $-\infty < \alpha < 1$ essentially because ex post substitution limits necessitate $\alpha < 1$.

Nakamura (2009) demonstrated that even if α is low initially there will be ex post substitution possibilities leading to a larger elasticity of substitution along the CES function as more of x_2 is available. That is, the ex post $\sigma = 1/(1-\alpha)$ may be high even if α is low initially. To understand this more explicitly assume that $\sigma^* = 0$. Firms substitute x_2 for x_1 ex post as p_1 increases and more of x_2 becomes available.

It is now useful to investigate the conditions under which $\sigma > \sigma^*$. Consider $f(\alpha) = \alpha^2 - \alpha + 1$. $f(0) = 1$ and $f_1(\alpha) = 2\alpha - 1$. Hence, $f(\alpha)$ attains a minimum value 0.75 when $\alpha = 0.5$. Consequently, σ is always greater than σ^*.[5] Some authors suggest that x_1 and x_2 are complementary in the range where $\alpha < 0$. See, for example, Pelli (2011). This is consistent with the ex ante specification of the elasticity of substitution alluded to above.

It may be suggested that there is no unique economic basis for the specification of a CES production function and that many variants may indeed be useful in different applications. Some innovative thinking is necessary to unravel all the possibilities.

Once the existence and nature of the elasticity of substitution is known it is not very difficult to specify the production function. However, a detailed specification of the sources of substitution has been elusive. It is generally argued that either technological features, as in Baumol et al (1982), or organizational aspects, emphasized in Sharkey (1982), provide the rationale behind the CES production function. A detailed specification of how these features account for the elasticity of substitution has been missing.

Lloyd (1989) offered a fairly useful lead though the following specification is not entirely based on his work. Suppose the firm uses only one input x_1 to produce y. Since diminishing returns can be expected the production function can be written as

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5 Ex post σ will be less than σ^* only in cases where factor prices do not change. Any possible change in factor prices ex post will provide the firm greater substitution possibilities.

$y = Ax_1^{\alpha}$; $0 \le \alpha \le 1$

The management may try to coordinate the use of resources to minimize or overcome the effect of diminishing returns. However, they may not succeed. In such a case the realized output may be

$$y = Ax_1^{\gamma} ;\ 0 \le \gamma \le 1$$
$$= A(x_1^{\alpha})^{\gamma/\alpha}$$

where γ represents organizational capabilities. Now, suppose a second resource x_2 is utilized in the hope that the effect of diminishing returns can be reduced. In practice x_1 and x_2 may cooperate and work together. Or, they may be competitive. For example, labor may be apprehensive about mechanization and refuse to cooperate. The output from x_1 and x_2 may then be

$$y = x_1^{\gamma} [a + b(x_2/x_1)^{\alpha}]^{\gamma/\alpha}$$

where $\alpha > 0$ if they cooperate and $\alpha < 0$ otherwise. Note that the problem of coordination persists even if it is not more severe. Hence, the resulting output can be expressed as

$$y = (ax_1^{\alpha} + bx_2^{\alpha})^{\gamma/\alpha}$$

In essence, the nature and extent of substitution will be determined by the cooperation that the management can elicit from x_1 and x_2. Rao (2011a) and Rao and Shrivastava (2011) considered other organizational dimensions which modify the productivity that can be elicited and the nature of this function. Similar results have been developed from a different perspective. In general, recent studies have been highlighting technological as well as organizational dimensions as they affect the elasticity of substitution and the production function. These specifications have yet to reach a steady state.

3. Estimation

Two broad patterns of estimating the parameters of the CES function are discernible. First, it is expected that the efficient choice of x_1,x_2 depends on y, the factor price ratio $p = (p_1/p_2)$ and the parameters of the production function. Hence ex post values can be utilized to estimate the necessary parameters. Several variants of this approach have been developed. Second, since the CES function is non-linear in parameters many estimable approximations have been conceptualized.

The original work of Minhas (1963) considered the following. The profit maximizing choice of x_1 by a firm will be such that

$$p_1 = ax_1^{\alpha -1} (ax_1^{\alpha} + bx_2^{\alpha})^{1/\alpha}$$
$$= a(y/x_1)^{1-\alpha}$$

The causation of p_1 determining the choice of x_1 is the correct interpretation. Hence, Minhas sought to estimate σ by using the equation[6]

$$\ln (y/x_1) = A + \sigma \ln p_1$$

[6] Crudely stated the idea behind such techniques is to find estimable linear relations based on the underlying economic process rather than utilize brute force numerical approximations.

The other reason for this choice was the difficulty in assembling data on capital (x_2). This procedure ran into many difficulties. Chief among them are the following. (a) Note that in the above specification A is a function of σ. This was neglected in the estimation. (b) Even the choice of x_2 contains information about σ. Neglecting it will make the estimator inefficient. (c) The σ obtained from the above equation is not free from dimensions of measurement of y and x_1 unlike the original theoretical construction. To circumvent some of these difficulties an equation

$$\ln p_1 = \ln a + (1-\alpha) \ln (y/x_1)$$

may be estimated. Two difficulties have been pointed out. (a) The direction of causation implied by the economics of choice necessitates treating (y/x_1) as an endogenous, and hence stochastic, choice. This gives rise to biased estimates of (1-α). (b) As Thursby (1980), Raval (2011) and others noted, even unbiased estimation of (1-α) does not guarantee the same for $\sigma = 1/(1-\alpha)$ since the pdf of the estimator of σ may not exist.[7]

By the nature of the profit maximizing choice of x_1,x_2 it is obvious that the choice of (x_1/x_2) depends on p. It is independent of y if the production function is homogenous. Further, the observed variations in y for given (x_1/x_2) will depend on x_2 and p. This is the nature of the expansion path. The entire information about the parameters of the CES function is contained in these two equations. The CES function is not, in itself, directly estimable. Formally, note that

$$p = (a/b)\ (x_1/x_2)^{\alpha}, \text{ and}$$

$$y^{\alpha} = bx_2^{\alpha}(1+p)$$

Hence, (a/b) and α may be estimated from the first equation and the second equation may be used to estimate b.[8] Some of the problems of estimation, as alluded to above, persist even if an unbiased estimator of α can be achieved.[9]

More recent studies make an attempt to overcome the dimension problem in a different way. The method essentially involves normalization such that the production function equation is satisfied uniquely for all values of α at the point of normalization. This was initiated and pursued in the work of La Grandville (1989) and associates. Consider the choice of $y_0,x_{10,}$ and x_{20} along with a+b = 1 and rewrite

$$(y/y_0) = [a(x_1/x_{10})^{\alpha} + b(x_2/x_{20})^{\alpha}]^{1/\alpha}$$

[7] Perhaps it would be reasonable to argue that the pdf can be defined except for a set of points of measure zero. Unbiasedness can be restored.

[8] Chetty and Sankar (1969) objected to these approaches on the grounds that the assumption of profit maximization in the choice of x_1,x_2 is dubious. Minhas (2008) also tends to accept this viewpoint. To the extent I am aware of it a better description is as yet not available.

[9] Note that, in general, x_{2t} differs from x_{2t-1} partly due to changes in p and also due to changes in y. Another approach suggests itself if we acknowledge that the effect of changes in y should be factored out. Assume that constant returns to scale prevail. Then, y_t/y_{t--1} is the scaling in x_1 and x_2 if factor prices do not change assuming that t represents time. Define $x_{2t}^* = [x_{2t} - (y_t/y_{t-1})x_{2t-1}]$ and similarly x_{1t}^*. These quantities exclusively capture the pure effects of elasticity of substitution. Hence, estimating the relation

$$(x_{2t}^*/x_{1t}^*) = B\ [(p_{1t}/p_{1t-1})/(p_{2t}/p_{2t-1})]^{\beta}$$

will yield β as an estimator of the elasticity of substitution. It appears that the problem with earlier methods of estimation was their inability to separate the scale effects and changes due to the elasticity of substitution appropriately. I am not aware of any empirical studies along these lines.

In practice, y_0,x_{10}, and x_{20} are chosen to be sample averages. This method cannot, however, circumvent many of the other problems mentioned above. Further, as Klump et al (2011) and Temple (2009) noted, all technology parameters will have a different interpretation when production technology changes. Thus, normalization cannot be a solution for all the problems encountered in estimation.

Models with many factors of production have run into other types of difficulties. Usually a Sato (1967) type nesting is suggested. However, as Kemfert and Welsch (2000) stated explicitly, the nesting, in itself, involves a prior judgment. In a nested CES, with capital, labor, and energy as inputs, two possibilities arise. Capital and labor may be treated as a composite index which trades off against energy. Alternatively it may be argued that energy utilization necessitates appropriate equipment implying that capital and energy are complements trading off against labor. This creates problems of getting different estimates of σ. Problems of estimation get compounded with various forms of generalization necessitated by the practical contexts studied.

At another extreme some econometric methods dwell on statistical and numerical approximations. They generally pay very little attention to the constraints prescribed by the economic choices underlying the data generation process. The following approaches have been prominent. (a) The method of non-linear least squares is adopted to estimate the CES function directly. It ignores the economics of the decision structure. Further, the numerical algorithms may not converge, may converge to a local minimum, and generally result in unreasonable parameter values. (b) Kmenta (1967) suggested a quadratic approximation of the CES. It results in non-linear restrictions on the parameters. It was also observed that the method results in large bias and mean square error. See, for instance, Corbo (1976). The translog approximations are related to this and have not done much better in practice. (c) Maximum likelihood methods did not fare much better in general. (d) Several numerical procedures have been suggested to circumvent problems of non-convergence and local minimum. A grid search, Newton gradient method, and global optimization algorithms have been attempted. (d) The Bayesian approach is also an iterative algorithm like all other numerical procedures. See, for example, Noda and Kyo (2011). Its superiority, if any, has not been convincing. As Henningsen and Henningsen (2011) and Kai et al (2010) pointed out, these algebraic approximations and statistical procedures tend to distort estimation significantly.

There is near unanimity that the estimation of the elasticity of substitution has been the least satisfactory. We do not even have adequate guidelines about the conditions under which one of these methods is superior to the others. This is a poor commentary on the state of non-linear econometric techniques.

4. Applications

The CES function has been considered as an important component in the analysis of many practical issues over the fifty years of its existence. However, some of the issues raised are of no practical relevance in contemporary economics. Even if this is true economic analysis does not seem to discard such hypotheses very easily. Similarly, despite its initial usefulness in explaining practical economic issues, the CES has been replaced by other and more insightful explanations. Thus, though the elasticity of substitution concept proved powerful in many applications it could not win over alternatives. Part of the reason is the lack of an economic basis for ex ante substitution, ambiguity in defining the CES with many inputs, and its inability to sustain the possibility that more than one choice of factor intensity or technology can be efficient ex post. Hence, though the CES was introduced as a way of closely reflecting empirical reality it failed on that count when it comes to accommodating other aspects of practical economic decision processes. The CES function remains an important dimension of economic models but only to the extent that it is complemented by equally relevant aspects of practical reality. Clearly, such an observation should not come as a surprise. The rest of this section will highlight these aspects of economic and econometric analysis. Due to limitations of competence and space I will only consider a few applications.

4.1. Factor Intensity Reversals

One of the earliest applications of the CES function was towards an explanation of the Leonteif paradox and the violation of the strict Heckscher-Ohlin theorem offered in Minhas (1962). The traditional assumptions were the following. (a) A country exports those commodities that require large amounts of its relatively abundant factor. (b) Relative abundance of a factor leads to a lower price. (c) Factor prices determine the commodities produced. (d) International trade equalizes factor prices across countries. In reality capital rich countries exported goods produced with relatively lower capital intensity. Utilizing the CES production function Minhas (1962) noted that

p = factor price ratio = $(a/b)\ x^{\alpha-1}$, where

$x = x_1/x_2$ = factor intensity

Hence, for any two countries j and k we have

$x_j/x_k = (bp/a)^{\sigma}$, where $\sigma = \sigma_k - \sigma_j$

Given $\sigma_j > \sigma_k$, $x_j/x_k < 1$ if $p < 1$. Factor intensity reversals will occur if $\sigma_j \neq \sigma_k$. This, Minhas argued, resolves the Leonteif paradox. No obvious reason was offered as to why the elasticity of substitution is different across countries or industries. It was invoked as an empirical fact.

A variety of questions have been posed subsequently. (a) Is the assumption that “factor prices are equal across industries or countries” valid? If not, differences in them can account for factor intensity reversals even if σ is the same. (b) Can factor prices be the sole determinant of technological development or its use? (c) What else, in addition to factor prices, determines the choice of capital intensity? Is it possible that factor endowments play a role and the effect cannot be fully captured by variations in factor prices? (d) What determines (b/a)? Can changes in this lead to factor intensity reversals? Dupuy and deGrip (2006) considered labor as non-homogenous and used it to explain patterns of trade. Brechet and Lambrecht (2010) considered the use of renewable or non-renewable resource use along with capital and labor. As it turns out, the capital intensity will reduce if the substitution between physical capital and resources is high. On occasions, capital and resource use (energy, in particular) may be complementary. Lin and Shao (2006) reported similar experiences with the use of information technology as a third resource.

On the whole, the hypothesis that differences in elasticity of substitution is the major source of factor intensity reversals cannot stand scrutiny in more general models. In addition, there have been doubts about according so much importance to factor intensity reversals in approaching major patterns of international trade. Unfortunately, as the study of Thompson (2011) exemplifies, economic analysis did not as yet discard this enterprise.

4.2. Growth Theory

A special issue of the Journal of Macroeconomics in 2008 revived interest in the neoclassical growth theory originally set out in Solow (1956). The basic departure was the use of the CES production function replacing the assumption of diminishing returns to factor use. It provided a possible explanation for differences across countries with respect to their growth paths.

In its essential detail, the Solow (1956) model can be stated in the form

$$dx/dt = sf(x) – nx$$

where x = factor intensity = x_1/x_2, $f(x) = y/x_2$, s = saving rate, and n = rate of growth of x_2. Similarly, dx/dt is the derivative of x with respect to time t. Four basic questions are of practical value. First, will $x = x(n,s)$ exist in the steady state where $dx/dt = 0$? Second, will the growth rate be independent of s? Third, will x_t converge to $x(n,s)$ as t progresses? In other words, is the long run equilibrium stable? Fourth, what are implications of the steady state for the functional distribution of income? Solow assumed constant returns to scale Cobb-Douglas production function. In that case

$$dx/dt = sx^a – nx \; ; \; 0<a<1$$

where a represents the share of x_1 in y. Diminishing returns to increases in x assure us that $x = x(n,s)$ exists in equilibrium, it is stable in the long run, and that the long run growth rate is n and hence independent of s. Hence, differences in growth rates across countries were often attributed to technical progress and differences in n.[10]

The starting point of the new argument was that when an economy experiences diminishing returns to the use of x_1 they can substitute x_2 for x_1 as a way out. In other words, the availability of substitution and its magnitude are the more important determinants of the equilibrium $x(n,s)$ and diminishing returns are incidental to it or they may be inconsequential. Consider the CES function as an alternative. Then, $f(x)$ can be written as

$$f(x) = (ax^{\alpha}+b)^{1/\alpha}$$

and the equilibrium condition is

$$s(ax^{\alpha}+b)^{1/\alpha} = nx$$

Observe that in equilibrium $x = x(n,s,\alpha)$ though the steady state rate of growth is still n.[11] If $\alpha > 0$, and consequently $\sigma > 1$, the equilibrium x and the productivity of x_2 will be higher. Antony (2007), Klump and La Grandville (2000), and Irmen (2010) noted this. Klump and La Grandville (2000) also argue that if $\sigma > 1$ a country will record higher output per unit of x_2 at every stage of its development (even outside the steady state). Antony (2007) and Klump et al (2011) also noted that σ tends to be low for poor countries and will be higher for rich countries even when $\sigma < 1$.

Turning to convergence and stability of the equilibrium x the general result is that $\sigma < 1$ is favorable. On the other hand, if $\sigma > 1$ the resulting instability indicates that an economy may

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10 Note that in recent years we have been saying that the significant economic growth of China and India is mainly due to their high saving rates. The neoclassical growth model is unequal to the task of explaining such trends even after incorporating the CES production function.

11 Suppose $\sigma<1$ and x is not very flexible. Then, given s the limitation on x may make some $n^*(\sigma) > n$ sustainable.

experience perpetual growth for certain initial states of factor intensity. The differences in the sources of convergence can be stated as follows. In the Solow model the assumption is that economies experience diminishing returns to increases in capital intensity. As a result, the rate at which x increases falls as x rises. This property assures convergence. In the CES version of the model $\sigma < 1$ indicates that substitution is not even possible at the same rate as x increases. Convergence emerges. This is not entirely convincing because the economic rationale underlying the emergence of $\sigma < 1$ is never clearly articulated. This is the primary weakness of models incorporating CES.

Let us now consider relative factor shares. Let s_1 and s_2 be the shares of x_1 and x_2 in y based on payments of marginal products. Then, clearly,

$$s_1/s_2 = (a/b)\ (x_1/x_2)^{\alpha}$$

Hence, factor shares and inequalities between different countries vary with α. This result, in essence, is similar to the factor intensity reversals argument of Minhas. Empirical evidence for this phenomenon is available in Aiyar and Dalgaard (2009).

Observe that

$$s_2 = b(x_2/y)^{\alpha}$$

Aiyar and Dalgaard (2009) argued that since b is a constant independent of α, the estimation of the elasticity of substitution will be efficient if we use

$$\ln s_2 = \ln b + \alpha \ln (x_2/y)$$

Further, this indicates that $s_2 < b$ if $\sigma < 1$. This reveals an anomaly in the Arrow et al (1961) interpretation of b. Recall that they designate it as the share of x_2 in y irrespective of the value of σ. As we noted earlier this interpretation holds only when $\alpha = 0$ and it is erroneous in all other contexts.

Note that we developed most of the promising results without invoking any technical progress. Hence, much of the work that tries to establish the importance of σ in growth theory is belabored.

The following additional arguments are significant. First, the work of La Grandville (1989) initiated a normalization procedure while defining the CES function. It turns out to be useful in efficient estimation and in establishing a strict positive relationship between output and the elasticity of substitution. Second, in more general models incorporating the CES, Papageorgiou and Saam (2008) noted that multiple equilibria are possible. This offers a basis to examine business cycle fluctuations. See, for example, Juillard and Villemot (2011). Third, many studies of La Grandville and others introduced technical change. The most important empirical finding is that the effect of differences in σ, on the differences in productivity, are far more significant compared to the differences that can be explained by technical change alone.[12]

On the whole, what may appear as spectacular progress with the introduction of the elasticity of substitution remains unsatisfactory because the economic content of the elasticity of substitution and the economic reasons that can explain differences across countries remain

[12] It is not altogether obvious that biased technical change may not, in itself, be a source of the observed elasticity of substitution. It is necessary to offer a convincing economic reason for why and how they are mutually exclusive.

largely an uncharted territory. Trying to establish the existence of differences in the elasticity of substitution empirically, taken in isolation, is hardly convincing.

4.3. Monetary Aggregates

Chetty (1969) was initially influenced by the notion of elasticity of substitution in the conceptualization of monetary aggregates. Apriori, currency and time deposits will differ with respect to their liquidity and the elasticity of substitution may be anywhere between zero and infinity. Hence, liquidity considerations, in addition to other characteristics of risk, create a certain utility from holding these monetary assets.[13] He argued that the CES function was useful in arriving at a monetary aggregate. Portfolio choice, it was argued, will be determined by the returns expected from the assets. The initial empirical work was encouraging. However, it is possible to argue that such an aggregate, on its own, would not be useful unless it is based on and/or can be shown to affect a crucial policy relevant variable. For, we do not estimate a production or utility function for its own sake. Its practical use ultimately determines relevance. Chetty's (1969) approach was not satisfactory.

At another extreme the convention was to use simple sum aggregates. They postulate that the components are perfect substitutes. However, different components of monetary assets, except currency, yield varying returns which cannot be considered as a flow of monetary services. Further, the return from all the assets is not the same. Hence, the real issue was to assign weights reflecting the money-ness of different assets. In Acharya and Kamaiah (1998), for instance, equal weights were replaced by currency equivalent quantities and the weighted sum was suggested as an aggregate. This does not address the issue of elasticity of substitution satisfactorily.

The Divisia monetary aggregate, which is a form of a normalized Cobb-Douglas function, appears to hold fort due to its flexibility in computation. See, for example, Acharya and Kamaiah (1998) and Ramachandran et al (2010). It is defined as follows. Let

Q_t = Divisia index at time t

x_{it} = quantity of the i^{th} asset

p_{it} = user cost of the i^{th} asset

$= (R_t - r_{it})/(1+R_t)$

r_{it} = return on the i^{th} asset

R_t = return on a benchmark asset

$s_{it} = p_{it}x_{it} / \sum p_{it}x_{it}$

$s_{it}^* = .5(s_{it} + s_{it-1})$, and

$\ln Q_t - \ln Q_{t-1} = \sum s_{it}^* (\ln x_t - \ln x_{t-1})$

13 Realistically it should be possible to define the elasticity of substitution without invoking the differences in return. Otherwise we must agree that the differences in liquidity are only due to the differences in return. The ex post definition of the elasticity of substitution, implied by Chetty's (1969) work, or any others that followed subsequently, is not clear about these nuances.

Though Q_t performed reasonably well in explaining inflation it remains a less than satisfactory aggregate due to its assumption of a unitary elasticity of substitution.[14]

There was a feeling that a useful definition of a monetary aggregate should be related to a specific aspect of macroeconomic functioning. Initially it was argued that such a monetary aggregate should explain changes in income in the best possible manner.[15] Subsequent studies indicate that core inflation, as a monetary phenomenon, would offer a more practical approach.[16] However, it became necessary to specify whose degree of substitution we are measuring and the channels through which each of the assets contribute to income or inflation and how the elasticity of substitution affects these channels. Very little information, if any, is available to guide us in the process of constructing the monetary aggregates. The additional complication is the possibility that the aggregates will be different depending on whether output or inflation is chosen as the guiding consideration.

The Phillips curve, despite all the skepticism about its existence, appears to resurface in such studies every once in a while. Montoro (2010), for example, demonstrated that oil price shocks generate an endogenous tradeoff between inflation and output stabilization when oil has low substitutability in production. The size of the endogenous cost-push shock generated by fluctuations in oil prices increases when oil is more difficult to substitute by other factors of production. There is hardly any consensus on the purpose that we expect a monetary aggregate to serve.

We may now reflect on what needs to be done to restore CES as a monetary aggregate. First, to compute an index at each point of time without repeating estimation, it should be possible to assign a value of α. The degree to which they are substituted between t-1 and t, or an ex ante measure such as the one suggested in section 2 above may come to our rescue. Second, s_{it} or s_{it}* are no longer satisfactory as measures of a and b. Some pragmatic choice must be suggested. These issues did not receive the attention that they deserve. The crux of the problem is in specifying the meaning and sources of ex ante substitution and the elasticity of substitution. We assume that substitution can be recognized since we experience it. However, it is necessary to revisit its economic content in specific contexts to make it practically relevant.

4.4. Product Variety

The consumers visualizing the possibility of substitution of products of different variety[17] in markets characterized as monopolistic competition is an accepted concept well before the

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14 There is a common refrain that such indices do not convey any economic meaning and that the reasons for why they are able to explain what they do (in this case inflation) are never really spelt out. It is indeed difficult to obtain any useful information if such a rigid posture is maintained. An optimal balance will remain elusive.

15 The celebrated St.Louis equation was of this nature.

16 As Das (2010) pointed out, money supply should then be looked upon as the sum of the liabilities of the banking sector and the government. Somewhat more generally it may be argued that the effect of supply side bottlenecks must be factored in while explaining core inflation so that we can separate the monetary and fiscal effects efficiently.

17 Assuming that each firm in the market produces only one product which is closely, but not perfectly, substitutable the market will have n substitutable varieties if n is the number of firms in the market. In the context of a multiproduct firm each of them can be characterized as producing m different products of different quality (durability etc).

arrival of the CES function. Dixit and Stiglitz (1977) formalized the characterization using the CES utility function. Let y_j; j = 1,2,…,n be the output of the j^{th} variety produced by firm j. Then, the utility function can be written as $g = (\sum y_j^{\alpha})^{1/\alpha}$. Let p_j be the price per unit of y_j. Assume that $C_j = F + cy_j$ is the cost of producing y_j and I = income of the consumer. Construct a price index $p = (\sum p_j^{1-\sigma})^{1/(1-\sigma)}$. It can be shown that in group equilibrium n can be determined by

$$I\ s[pn^{1/(1-\sigma)}]/pn = F(\sigma - 1)/c$$

Clearly, n is a function of σ. The issue was the determination of an equilibrium value of n to verify the possibility that the n obtained under group equilibrium is greater than that under competition. Chamberlin predicted that product proliferation will occur.

A large number of theoretical studies examined this and related issues. Most of them have been summarized in Chang (2011). In general, it was concluded that there may be too much or too little product variety under market equilibrium compared to the social optimum. First, under competitive conditions prices will be close to marginal cost. In turn, this will be lower than the average cost if increasing returns to scale obtain. In such a case revenues will not be sufficient to cover the cost of some socially desirable products. They will not be produced. Product proliferation cannot occur. On the other hand, the monopoly power of each firm makes price exceed the marginal cost. This may still generate proliferation. In general, lower values of the elasticity of substitution, lower fixed costs and/or decreasing returns to scale, and high values of income have been considered as factors contributing to a proliferation of varieties.

However, the following major shortcomings of the CES function approach have been recorded. First, price differences are not the only (or major) factor governing consumer choice. Differences in the elasticity of substitution, as perceived by different individuals, explain the observed intensity of choice. This argument is similar to the factor intensity reversals alluded to by Minhas (1963). Second, the CES function rarely justifies anything but only one choice of variety by a consumer. However, even casual observation indicates that a consumer may prefer a number of substitutable products. It is difficult to introduce this feature in related models. Third, while these models account for the substitution effects on consumption and prices in excess of marginal costs they rarely model the effect of the elasticity of substitution on the costs of production. Fourth, many firms tend to be of a multiproduct nature. Very little analytical work is available along these lines. A strand of thought based on the work of Raubitscheck (1987) has been influential. However, in the context of multiproduct firms many considerations, other than the elasticity of substitution, are prominent. See, for instance, Bernard et al (2010), Goldberg et al (2011), and Soderbom et al (2011). Fifth, the elasticity of substitution is a measure of the elasticity of demand and it determines the price cost margins. Higher elasticity of substitution may have the effect of decreasing or increasing the number of products depending on which of these forces is dominant.

Empirical studies have been of two broad types. First, retail stores and the variety and quality they offer has been a subject of a large number of studies. Second, it is generally believed that the reduction in varieties, if it manifests in markets that can be characterized as monopolistic competition, can be overcome by international trade. Hence, the elasticity of substitution between domestic and tradable goods has a bearing on welfare gains. Hence, the CES function was used extensively in studies of international trade.

Consider the studies on retail stores. Perforce they are multiproduct in nature. Continuous versions of the CES were necessary to deal with the optimality of such varieties and qualities being offered. From a consumer viewpoint the optimality of a large retail stores depend on the volume of purchases per visit, frequency of purchases, and the transportation costs of visiting the stores that together constitute transaction costs. A store may, however, derive some monopoly power since providing greater variety may be advantageous once consumers are accustomed to it or find it easier to visit it. Richards and Hamilton (2006) also found it necessary to model substitution of products within a firm and across stores. In general, estimating market share equations by non-linear three stage least squares appeared to offer reasonable estimates of within store substitution across varieties and between stores. Their conclusions, that heterogeneous offering by one firm leads to greater variety but few firms and heterogeneous offerings by different firms leads to a larger number of firms but few varieties, are consistent with empirical facts. In a somewhat different vein Wuergler (2010) found that higher income inequality resulted in higher product varieties being offered. Appropriate variations in the CES specification combined with other economically relevant empirical facts appear to have a prospect of doing better.

Models of international trade are distinctly different. First, as Feenstra (2010) noted, the welfare increasing effects of imports may be neutralized if they substitute for domestic varieties and there are losses (in income and employment) due to changes in domestic production. Lai and Trefler (2002) also observed that gains from liberalization of trade depend on the elasticity of substitution between domestic and traded goods. One drawback of such CES function based models is that they model the substitution effects on consumption but fail to account for the substitution effects on production. This was noted in Lai and Trefler (2002). They also noted that the estimates of elasticity of substitution fluctuated wildly between different studies of this vintage. They have not been convincing because the estimated welfare gains, based on compensating variation, are very sensitive to the estimated elasticity of substitution. Further, the difficulties of estimation are compounded because the demand curves implied by the CES based specifications involve complex price terms that depend on the elasticity of substitution. Inefficient estimation of the elasticity of substitution has been a bother as well. Their study also pointed out the perils of careless use of data. In general, the available data relates to the trade between a country X and each of a,b, and c. Usually the analysis of such data estimates the elasticity of substitution between (X,a), (X,b) etc using a cross section. The estimated values are often quite large. However, notice that the relevant elasticity of substitution is for trade between X and a and so on. A few estimates suggest that the elasticity of substitution based on such time series data is rather low. Callous use of data and disregard for the economic processes that generated it has been a constant source of error. This has lead to the observation of Lai and Trefler (2002) that the conclusions of most studies, based on such analysis, are driven by models but not the data.

4.5. Organizational Synergies

There has been an extensive documentation of the emergence of spinoff firms and the reasons for innovators making such decisions over the past fifteen years. They exhibit a bewildering variety of patterns. Many studies, that empirically identify the patterns as well as theoretically explain them, have emerged. Klepper (2009) offered a recent survey of the issues at stake. Three prominent economic mechanisms have been identified as the triggers for spinoff, viz., the market interaction (and the elasticity of substitution between products that a firm produces), organizational capabilities of the management, and financial arrangements. As a general rule, as Klepper and Thompson (2010) hypothesized, there will be disagreements at the apex of the spinoff decision.

Unfortunately, there has been no clarity regarding the nature of the products, as indicated by the elasticity of substitution, that spinoff for one or more reasons alluded to earlier. In particular, Klepper and Sleeper (2005 p.1293) observed that “spinoff firms pursue ideas involving new niche markets or technologies their parent firms are unwilling or slow to pursue. Industry conditions favorable to the creation of niche markets are thus conducive to spinoff.” That is, spinoff of products with low elasticity of substitution occurs more frequently. In sharp contrast to this Klepper (2009, p.163) pointed out that “initially spinoff firms tend to produce types of products that were a substitute of those produced by their parent firms.” For, new product introduction may cannibalize the demand for existing products. Turning to organizational capabilities Hyytinen and Maliranta(2008) observed that they will increase with the elasticity of substitution simply to constrain divisional managers from free riding on the effort of others. In general, studies that emphasize organizational capabilities, suggest that highly substitutable new products will experience a spinoff. On the other hand, new and distinct products with a low elasticity of substitution may require different, though not necessarily a higher quantum, of organizational capabilities. The incumbent management’s lack of such skills triggers a spinoff. Cassiman and Ueda (2006) found this rather surprising. The conflict is similar even in the context of raising adequate finances. Boreiko and Murgia (2010) contend that equity holders may find it difficult to ascertain the profitability of a new product if it has a low elasticity of substitution with existing products. On the contrary, Chemmanur and Yan (2004) noted that a large elasticity of substitution, given its cannibalization implications, will reduce the market value of the new product and make it difficult to attract finances.

The management literature did not make an attempt to examine the channels through which the elasticity of substitution has an effect on the spinoff decision. Fundamentally, an organizational production function concept was missing. In a series of recent studies Rao (2011 a,b) and Rao and Shrivastava (2011) made attempts to redress this balance using the CES function. Basically they argue that substitution possibilities arise from consumer choice and that a firm tends to be successful only if their organizational capabilities can accommodate this. Further, they argue that the returns to scale parameter would best represent the synergies that organizational capabilities offer. Similarly, they incorporate managerial preferences and financial synergies in an extended CES function. Given this basis it is easy to suggest that the firm will integrate a new product only if all stakeholders gain. The following spinof patterns have been identified on this basis. Organizational capabilities limit the introduction of products with low elasticity of substitution. High elasticity products spinoff mainly due to inability of financial markets to provide the requisite finances since valuation problems arise. Such an analysis is important because it can resolve the ambiguity that arises when only one source of spinoff is acknowledged.

These studies are still quite new though they once again demonstrate the significance of elasticity of substitution and the CES formulation in explaining certain empirically observed phenomena. Much progress will be necessary to judge the robustness of this conceptualization.

4.6. Human Development Index

In more recent literature there have been efforts to compare the relative development of different countries, and/or regions of a country, on the basis of their achievements in income, environment, health care, education and so on. An increase in any one of them improves welfare but only to the extent others are also available in requisite measure. Equivalently, a society may view with equal equanimity an increase in one of the measures while some other measure remains lower. The construction of HDI attempts to capture this substitution and the share of contribution of each of the measures to overall welfare.

Clearly, the construction of a HDI can be formulated in the framework of a CES function. However, the substitution and distribution parameters are more of a normative judgment instead of something estimable from available data. Nathan and Mishra (2010), for instance, postulated that societies seek fixed proportions of each (their line of equality) and that deviations from the norm reduce welfare. Thus, they arrive at a HDI

$$H = 1 – [(1\text{-}y)^{\alpha} + (1\text{-}h)^{\alpha} + (1\text{-}e)^{\alpha}]^{1/\alpha}/3$$

where y = income, h = health care, and e = education

They examined empirically the implications of choosing α =1 or 2. Their results suggest that α = 2 provides a more satisfactory measure since it "penalizes countries that give greater emphasis to one dimension while neglecting other dimensions." – p.212.

Defining an ideal level for each of the measures, their relative weights in welfare, and the degree of substitution need a great deal more of investigation. This work has just begun. By the nature of the problem it may be difficult to say that all countries accept the same normative values for these parameters and/or that they remain the same for even one country over time. Some simple ex ante conceptualization is necessary so that it is measurable without having to depend on ex post estimation. Recall that a similar problem was encountered while defining monetary aggregates.

While closing this section it is important to record the following. A pertinent question throughout this section has been: what is the source of the substitution ex ante? That factor price variations induce substitution ex post is not an answer.[18] For, it comes about ex post only if it is possible ex ante. An answer that some technologies are, by their very nature, capital intensive while others are not begs the question instead of explaining why such ex ante choices are possible. There is no good answer to this basic question despite all the scholarship that we experienced over the past fifty years. Econometric analysis per se can be hardly expected to resolve such tangles. It is possible to argue that R&D and capital intensive technology is itself induced by a desire to minimize costs by reducing the use of expensive and scarce labor and/or by increasing the productivity of labor. This may explain the ex ante emergence of substitution. The problem is that it opens a new question: is technical progress a function of the elasticity of substitution instead of these two being independent phenomena? See footnote 6 above.

In general, empirical reality is far too complex for it to be encapsulated in simple economic or algebraic generalization. Eisner and Strotz (1968, p. 465-6) presented such a

[18] Ironically, capital intensive technologies have been the preference even in labor abundant economies. Hence, factor price variations alone cannot explain the observed substitution either ex ante or ex post. Some other necessary explanation is missing from the analysis.

dilemma way back in history. As they put it, "when theory and data are in conflict, shall we trust whatever insights … we can get from our micro theory – the statistics be damned – or shall we reject our a *priorism* and casual empiricism in favor of hard-headed claims about what the figures show?" Also note that for any given activity, say transportation between two cities, many technologies are utilized given the factor prices and macro level factor proportions. Something other than factor price variations will be necessary to explain this. A CES function per se cannot be trusted as the only empirically useful guideline in all practical contexts. All said and done every new concept has limited practical use in empirical reality. However, there is tendency to abuse it uncritically. I am afraid the CES function has been a victim of such callous use. Given the present day attitudes in economic research an appropriate course correction is unlikely.

5. Conclusion

Though I have been very selective in my presentation it is obvious that the CES production function was important in many applications and was instrumental in providing some incisive insights into the economics of decision making processes and its implications. However, as it is generally true of economic analysis of recent vintage, we tend to utilize otherwise useful concepts without paying attention to the economic processes at work that gave rise to observed data. More pointedly, we may claim that algebraic and statistical intricacies dominate the economic content. The urge for and rush to publish resulted in excessive turbulence. We hardly seem to look for ways of getting out of the vortex.

Econometric methods have not fared better. Early pioneers repeatedly alluded to the need for enveloping economic decision processes into our estimation. As Hicks (1965, p.4) argued "we have no right to conclude, from the mere existence of the statistical device, that the economic forces making for trend and fluctuations are any different, so that they have to be analyzed in different ways. It is inadvisable to start our economics from the statistical distinctions, though it will have to come in at the appropriate point, as an instrument of application." This has not been adhered to in econometric practice. The work of Minhas started with an economic basis to estimate the parameters of the CES function. It was inadequate but subsequent developments totally ignored this requirement.

Theoretical implications of the elasticity of substitution have been easy to develop. Empirical estimation and use in empirical and policy use have not been equally successful. The necessity for and the lack of an ex ante estimation method without going through elaborate econometric procedures is at the apex of the failure. It is ironic that what was considered as empirically obvious has not been equally useful in empirical application and use. We need a clear statement on the sources and definition of the ex ante elasticity of substitution and a simple yet robust estimation procedure for it to become a useful tool for econometric practice in realistic contexts.

Some specifications and methods of estimation could yield satisfactory estimates of the elasticity of substitution while other failed. It is not as yet clear that inferences can be drawn about the empirical contexts in which the use of CES is useful or about the efficient method of estimation in each such context. Similarly, when an application was not successful we wish to know if the modeling is faulty, the use of CES was inappropriate, or the estimation procedure is itself the source of the problem.[19] At the present stage of development we can hardly be expected to offer any usable guideline.

Simple fundamental insights into practical economic problem solving appear rarely. The CES function is surely one such. Subsequent herd like behavior has been too mechanical despite sincere attempts to provide economic insights into a variety of decision processes. However, it is obvious that the idea about elasticity of substitution is very important in practical reality. It is necessary to come to grips with the explanation of its sources and empirically measure it efficiently. Neither of these has been achieved satisfactorily so far. A breakthrough is very much in order. It may take another genius like Minhas to provide the necessary impetus. The wait is certainly worth it.

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[19] Perhaps the relevant questions can be summarized as follows. Are the issues raised in the context of a specific application important in the practical context? Have they been modeled appropriately by using the CES function? Is it possible that the elasticity of substitution is not the most important dimension while constructing an answer? Can we be sure that the data from which estimation has been flagged off really the most appropriate in the context of the model at hand? What is the most efficient method of estimation based on experiences with similar contexts? A methodological study dealing with some comparative assessments is in order.

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