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PRODUCTIVITY DIFFERENTIAL AND BILATERAL REAL EXCHANGE RATE BETWEEN INDIA AND US

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Abstract

Using annual data for 1959-2001 for India and USA, we test for the presence of real effects on the equilibrium real exchange rate (the Harrod Balassa Samuelson effect) in a non-linear framework. The real exchange rate is modelled as an exponential smooth transition autoregressive (ESTAR) process where we model the equilibrium real exchange rate as dependent upon differences in real income per capita. We find that higher productivity growth in US is accompanied by appreciation of its real exchange rate vis-à-vis India. We find significant evidence of non-linear mean reversion towards the long run equilibrium. We analyse the non-linear impulse response functions and find the evidence of faster mean reversion with larger shocks.

Keywords: Real exchange rate, productivity differential, Balassa Samuelson effect

JEL classifications: F31, E23

1.Introduction

Within the Balassa-Samuelson framework the present paper aims to examine the effects of productivity differential on the bilateral real exchange rate between India and the United States. The novelty of the paper is that unlike any previous study on India, we examine this effect employing the exponential smooth transition autoregressive (ESTAR) model. Given the clearly upward trend in relative productivity between the USA and India over the past few decades this seems worthwhile.

The paper is developed as follows. Section 2 briefly reviews the salient literature while section 3 explains the methodology. Section 4 discusses and analyses the results and the final section concludes.

2. Long-run PPP and its Dynamics

Equilibrium models of exchange rate determination in presence of transaction costs have been proposed by Benninga and Protopapadakis (1998), Dumas (1992) and Sercu, Uppal and

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Van Hull (1995). As a result of cost of trading goods, persistent deviations from PPP are implied as an equilibrium feature of these models. Deviations are left uncorrected as long as they are small relative to the cost of trading.

Deviations from PPP are shown to follow a non-linear process that is mean reverting, with the speed of adjustment toward equilibrium varying directly with the extent of deviation from PPP. Within the transaction band, when no trade takes place, the process is divergent so that exchange rate spends most of the time away from parity. This implies that deviations from PPP may last for a long time, although they certainly do not follow a random walk. In recent years, there has been a surge of papers econometrically analysing the nature of PPP dynamics in a nonlinear framework. Taylor, Peel and Sarno (2001) provide strong confirmation that four major real bilateral dollar exchange rates are well characterised by smooth transition autoregressive (STAR) models. Because of the non-linearity nature of the estimated models, the half-lives of shocks to the real exchange rate vary both with size of the shock and with initial conditions. For dollar-mark and dollar-sterling in particular, even small shocks of one to five percent have a half life under three years. Some attempts have been made recently to incorporate the determinants of the equilibrium real exchange rate in linear models of the adjustment of real exchange rates. Engel and Kim (1999), Lothian and Taylor (2004) employ per capita real incomes motivated by the models of Balassa (1964), Samuelson (1964) and Lucas (1982). In contrast to studies for developed countries like the UK, USA, Japan, there have been limited studies for India. To a large extent, most of these studies have examined the behaviour of real exchange rate of India in a linear framework, for example, Weliwita (1998), Kholi (2002) and without reference to impacts on the equilibrium real rate.

3. Methodology and Estimates

We have checked the non-linearity property of the Indian real exchange rate vis-à-vis US (1959-2001) using Brock, Dechert and Scheinkman (BDS) Test. A powerful test used for independence and under certain circumstances for non-linear dependencies- was developed by Brock, Dechert and Scheinkman (1996) and is based on the correlation integral. The BDS statistic tests the null hypothesis that elements of a time series are independently and identically distributed (IID).

For a time series which is IID, the distribution of the statistic $w_m(\varepsilon) = \dfrac{\sqrt{n}\{C_m(\varepsilon) - C_1(\varepsilon)^m\}}{\sigma_m(\varepsilon)}$ is asymptotically N(0,1). $w_m(\varepsilon)$ is known as BDS statistic. $C_m(\varepsilon)$ denotes the fraction of m-tuples in the series which are within a distance of each other and $\sigma_m(\varepsilon)$ is an estimate of standard deviation under the null of IID. The test statistic is asymptotically standard normal under the null of whiteness. The null is rejected if the statistic is absolutely large (say greater than 1.96).

If the null hypothesis of IID cannot be accepted this implies that residuals contain some kind of hidden structure which might be non-linear or even chaotic. Following the recommendation by Brock, Hsich and LeBaron (1991) and suggestions by Brooks and Heravi (1999), we set $\dfrac{\varepsilon}{\sigma}$ = 0.5 to 2 and m= 2 to 4.

First we determine the prewhitening model AR(p). The values of p from 0 upto 4 are considered and the one with minimum SC criterion is chosen. The best AR model in this case is AR(1). The next step is to save the residual of the best linear AR(p) model that is AR(1) and test for any remaining serial dependence. Given the limited sample available the tests are conducted using both asymptotic theory and bootstrap. The values under asymptotic theory are based on large sample distribution of the relevant test statistic. For the bootstrap results 1000 new samples are independently drawn from the empirical distribution of the prewhitened data. Each new sample is used to calculate a value for test statistic under the null hypothesis of serial independence.

The obtained fraction of 1000 test statistic which exceeds the sample value of test statistic from original data is then reported as the significance level at which null hypothesis can be rejected. If the null hypothesis of IID cannot be accepted, this implies that residuals contain some kind of hidden structure which might be non-linear.

P-values of less than 0.05 suggests rejection of linearity. Table 1 shows the BDS test results. The BDS test suggests that the Indian real exchange rate vis-à-vis US (1959-2001) is non-linear. So we use non-linear model to determine Indian real exchange rates.

Table 1. BDS test results

	Bootstrap		
	$\frac{\varepsilon}{\sigma}=0.5$	$\frac{\varepsilon}{\sigma}=1$	$\frac{\varepsilon}{\sigma}=2$
$m=2$	0.001	0.000	0.001
$m=3$	0.002	0.000	0.002
$m=4$	0.002	0.001	0.000
	Asymptotic theory		
$m=2$	0.000	0.001	0.000
$m=3$	0.001	0.001	0.000
$m=4$	0.001	0.002	0.001

Only p-values are reported.

In this paper the real exchange rate is modelled as an ESTAR process in which the equilibrium real exchange rate is dependent upon differences in real income per capita. (proxy for Harrod Balassa Samuelson effect)

$$y_t = \alpha + \delta x_t + e^{-\gamma (y_{t-1}-\alpha-\delta x_{t-1})^2}\left[\sum_{i=1}^{n}\beta_i (y_{t-i}-\alpha-\delta x_{t-i})\right] + u_t \quad \text{... (1)}$$

$$y_t = s_t + p_t - p_t^*$$

where y_t is the real exchange rate, s_t is the logarithm of the spot exchange rate (Indian rupees / US dollar), p_t is the logarithm of US price level, p_t^* is the logarithm of Indian price level, α is a constant, x_t are the determinants of the equilibrium level of real exchange rate, γ is a positive constant, the speed of adjustment, β_i are constants and u_t is a random disturbance term. Here

x_t is defined as productivity differential while productivity differential is measured as log difference of real GDP per capita of the two countries. Given the data availability, we measure the productivity term as the ratio of total national output, real GDP, to total population.

We have used ESTAR model and the importance of ESTAR model in this particular context is the following. One particular statistical characterisation of non-linear adjustment which appears to work well for exchange rates is the smooth transition auto-regressive (STAR) model.

A smooth transition model has the form

$$y_t = \sum_{i=1}^{p} \alpha_i y_{t-i} + \ 1 - F_t \ \left\{ \sum_{i=1}^{r} \delta_i y_{t-i} \right\} + u_t \qquad \text{... (2.1)}$$

where α_i and ∂_i are constants and F_t is the continuous transition function, which is usually specified to be bounded between zero and unity. In the STAR model adjustment takes place in every period but speed of adjustment varies with the extent of the deviation from parity.

Ozaki (1978) and Hagan and Ozaki (1981) introduced the exponential auto-regressive model known more widely these days as the exponential STAR (ESTAR) model.

In the ESTAR model

$$F_t = e^{-\gamma \ y_{t-d} - \lambda \ 2} \qquad \text{... (2.2)}$$

where d is the delay, γ is a positive constant and λ is a constant.

The transition function for the ESTAR model is symmetric in deviations of y_{t-d} from λ. The transition function determines the degree of mean reversion. The parameter γ determines the speed of transition process between two extreme regimes. We observe from (2.1) that as F_t varies between zero and one we obtain an infinite number of different auto-regressive processes each corresponding to a different state.

An interesting special case of ESTAR which illustrates some of the possibilities is the model

$$y_t = y_{t-1} e^{-\gamma \ y_{t-1} \ 2} + u_t \qquad \text{... (2.3)}$$

When the deviation y_{t-1} is large (2.3) will give approximately $y_t = u_t$

Conversely when y_{t-1} is small (2.3) will give approximately $y_t = y_{t-1} + u_t$

or $y_t - y_{t-1} = u_t$ so that y exhibits behaviour which varies from where y is approximately random to where changes in y are approximately random.

We can observe the non responsiveness of $y_t - y_{t-1}$ to y_{t-1} near zero, which is the equilibrium value in this model. This type of adjustment was employed by Michael, Nobay and Peel (1997) to model deviations of real exchange rates from equilibrium. It captures the idea that there is little response of real exchange rates to deviations from equilibrium when they are small but adjustment is proportionately faster the greater the deviation.

We estimate equation (1) using annual data (obtained from International Financial Statistics) for the period 1959 -2001 as monthly data for GDP are not available for India. We have checked the stationarity property of the variables.[2] The results of the estimation are summarised in Table 2A. This clearly suggests that the coefficient of x_t is significantly positive. The latter in turn provides support to the central Balassa -Samuelson hypothesis that higher productivity growth in US is accompanied by appreciation of its real exchange rate vis-à-vis India.

Table 2A. Estimation of ESTAR model (equation 1)

Estimated model: $y_t = \alpha + \delta_1 x_t + e^{-\gamma \; y_{t-1} - \alpha - \delta_1 x_{t-1} \; 2} \; y_{t-1} - \alpha - \delta_1 x_{t-1}$ [using annual data from 1959-2001]

α	δ_1	γ	R^2	*Standard error of regression*
0.5 (0.18)	0.55 (0.04)	1.53 (0.12)	0.96	0.06

Figures in first brackets are standard errors.

Table 2B. Test for serial correlation in the residual of equation 1

Breusch-Godfrey Serial Correlation LM Test:			
F-statistic	1.161571	Prob. F(2,37)	0.324146
Obs*R-squared	2.477197	Prob. Chi-Square(2)	0.289790

Table 2C. Test for heteroskedasticity in the residual of equation 1

White Heteroskedasticity Test:			
F-statistic	0.836022	Prob. F(6,35)	0.550531
Obs*R-squared	5.264814	Prob. Chi-Square(6)	0.510325

Productivity differential is considered as the major determinant of the real exchange rate as we check the Balassa Samuelson effect (following Lothian and Taylor, 2004). The effects of other determinants are captured in the residual of equation (1).

The reported tests for serial correlation and heteroskedasticity for equation (1) suggest that serial correlation and heteroskedasticity do not exist in the residuals (Table 2B and 2C).

[2] We performed two sets of unit root tests, namely, Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests. We note that null of unit root cannot be rejected for y_t or x_t.

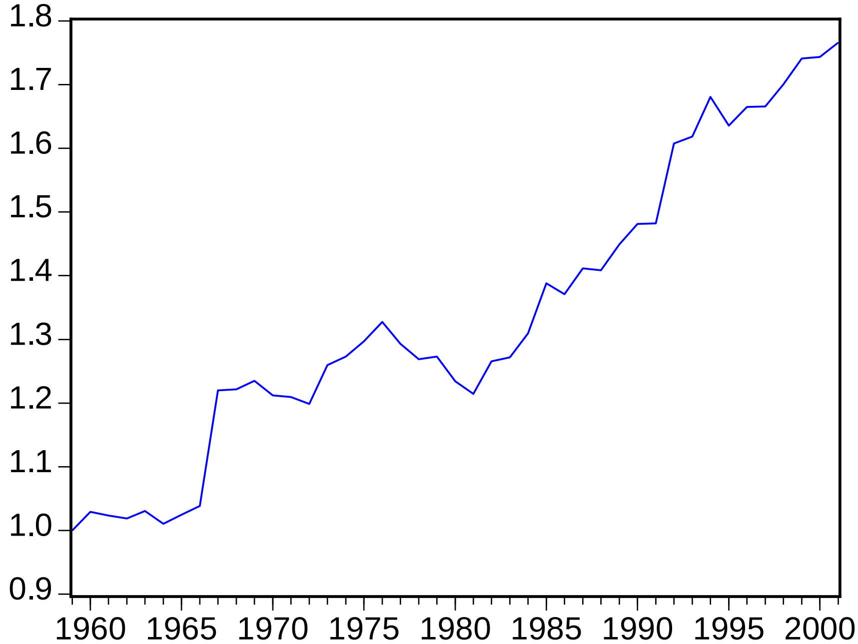

Figure1. Plot of productivity differential (x_t)

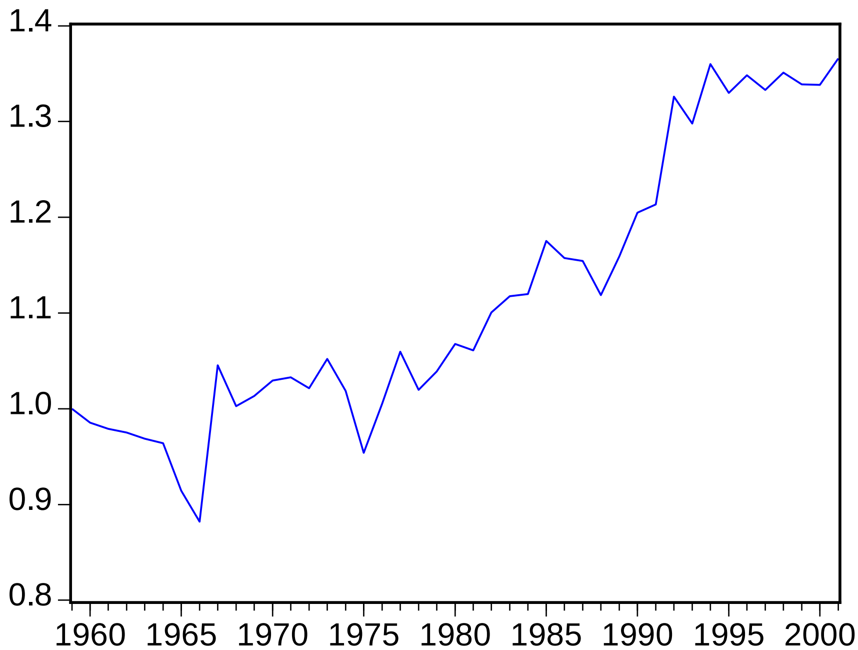

Figure 2. Plot of real exchange rate (y_t)

In September 1975 the Rupee's ties to the Pound Sterling was broken and India conducted a managed float exchange rate regime with the rupees effective rate placed on controlled floating basis and linked to a basket of currencies of India's major trading partners. India adopted market determined exchange rates from 1993. Though India adopted market determined exchange rates from 1993, productivity differential had some sort of influence on exchange rate before 1993 also. Though this model has much more relevance for the post liberalisation period, it has some relevance for the pre-liberalisation period also.

Using standard methodology as set out in e.g. Taylor et al (2001) we report half-lives of shocks in Table 3. For shocks of five per cent or three per cent, the half life is two years , while larger shocks like ten per cent or twenty per cent have a half life of one year. These results therefore accord broadly with those reported in Taylor, Peel and Sarno (2001) and shed some light on Rogoff's (1996) PPP puzzle. Once non-linearity is allowed for, even small shocks of one to five percent have a half life of two years or less, conditional on average history and for larger shocks the speed of mean reversion is even faster.

Table 3. Estimated half-lives shocks in years for Indian Rupees / US dollar (1959-2001)

Estimated γ	3% shock	5% shock	10% shock	20% shock
1.53	2	2	1	1

Concluding comments

This paper has examined the effects of productivity differential on bilateral real exchange rate (Harrod Balassa Samuelson effect) between India and US, in the context of a non-linear adjustment of real exchange rates to their long run equilibria.

We find evidence of Harrod Balassa Samuelson effect using annual data for the period 1959-2001. The impulse response functions[3] for shocks of various magnitudes to the real exchange rates suggest that the half lives for large shocks of twenty per cent or ten percent are only one year. For small shocks like three per cent and five percent they are two years. Our

[3] We must take into account that a number of properties of the impulse response functions of linear models do not carry over to the non-linear framework. In particular, impulse responses produced by non-linear models are history dependent, so they depend on initial conditions. They are dependent on the size and sign of current shock and they depend on future shocks as well. Koop, Pesaran and Potter (1996) introduced the generalised impulse response functions (GIRF) for non-linear models. The GIRF is defined as the average difference between two realizations of the stochastic process (y_{t+h}) which start with identical histories up to time t-1 (initial conditions) The first realization is hit by a shock at time t while the other one is not:

$$GIRF_h(h, \varphi, m_{t-1}) = E(y_{t+h} \mid u_t = \varphi, m_{t-1}) - E(y_{t+h} \mid u_t = 0, m_{t-1}) \quad \dots (3)$$

where h = 1, 2,….. denotes horizon, $u_t = \varphi$ is an arbitrary shock occuring at time t and m_{t-1} defines the history set of y_t. Given that φ and m_{t-1} are single realizations of random variables, (3) is considered to be a random variable. In order to obtain sample estimates of (3), we average out the effect of all histories m_{t-1} that consist of every set ($y_{t-1}, \dots, y_{t-p}$) for $t \geq p+1$ where p is the autoregressive lag length and we also average out the effect of future shocks u_{t+h}. We set φ = 3%, 5%, 10%, 20%. The different values of φ would allow us to compare the persistence of very large and small shocks.

The half-lives of shocks is the time needed for $GIRF_h < (1/2)\varphi$. Non-linear impulse response functions illustrate well the non-linear nature of the estimated real exchange rate models, with larger shocks mean reverting much faster than smaller shocks.

results support the earlier empirical findings of Lothian and Taylor (2004) and Paya and Peel (2004) that the rate of mean reversion is much faster with larger shocks.

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