

## A BAYESIAN EXTENSION OF THE J-TEST FOR NON-NESTED HYPOTHESES

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### Abstract

*It is a common practice to use the Davidson and MacKinnon's J-test in empirical applications to test non-nested model specifications. However, when the alternate specifications fit the data well the J- test may fail to distinguish between the true and false models: the J-test will either reject, or fail to reject both specifications. We show that it is possible to use the information generated by the J-test and combine it with the Bayesian posterior odds approach that would yield an unequivocal and acceptable solution for non-nested hypotheses. We show that the approximations of Schwarz and Bayesian Information Criterion based on classical estimates for the J- test yield the Bayesian posterior odds without any need for the specification of the prior distributions and the onerous Bayesian computations.*

**Keywords:** Non-Nested Hypotheses, J-Test, Log Likelihood, Posterior Odds, SIC, Bayes Factor

**JEL Classifications:** C52, C11, C12

### 1. Introduction

The non-nested tests of hypotheses arise in situations when the alternate hypothesis cannot be derived as a special case of the null hypothesis. This may arise either due to completely different sets of regressors in competing model specifications or different distributions of the stochastic terms. One of the most widely used tests in applied econometrics for comparing non-nested hypotheses is the J-test proposed by Davidson and MacKinnon (1981).

When each of the competing hypotheses is successful in explaining the variations in the data, the J-test may not be able to discriminate between alternative specifications. Some of the situations in which the J-test does not discriminate between the competing specifications have

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already been noted. Godfrey and Pesaran (1982) state the following one or more conditions where the J-test is likely to over reject the true hypothesis: (i) a poor fit of the true model; (ii) low or moderate correlation between the regressors of the two models; and (iii) the false model includes more regressors than the correct specification. Davidson and MacKinnon (2004) agree that the J-test will over-reject, “often quite severely” in finite samples when the sample size is small or where conditions (i) or (iii) above are obtained. Gouriéroux and Monfort (1994) conclude that the test is very sensitive to the relative number of regressors in the two hypotheses; in particular, the power of the J-test is poor when the number of regressors in the null hypothesis is smaller than the number of regressors in the alternative one.

It is possible, however, to find examples in the literature where none of the above noted conditions are violated<sup>4</sup> and where the J-test rejects all models. McAleer’s (1995) survey of the use of non-nested tests in applied econometric work reports that out of 120 applications all models were rejected in 43 applications. However, he did not break down the rejections by the type of test used. Bernanke, Bohn, Reiss (1988) had similar conclusions of non-rejection of two non-nested aggregate investment demand models.

Here, we give three examples of empirical work on the consumption functions that illustrates this situation.

Greene [2003, 2007] reported the results of comparing the same two consumption function hypotheses using quarterly data for the period 1950:2 – 2000:4. The results of the test lead him to a similar conclusion: “Thus,  $H_0$  should be rejected in favor of  $H_1$ . But reversing the roles of  $H_0$  and  $H_1$ ...  $H_1$  is rejected as well.” Although Greene did not report on the goodness of fit, it is very likely, as in the E-Views 5 data, that each of the models had explained almost all of the variation in consumption.

In the econometrics software E-Views the J-test is used to compare two hypotheses regarding the determinants of consumption. The first hypothesis is that consumption is a function of GDP and lagged GDP. The alternative expresses consumption as a function of GDP and lagged consumption. The data used are quarterly observations, 1947:2 – 1994:4. The conclusion reads: “we reject both specifications, against the alternatives, suggesting that another model for the data is needed.” [p. 581]. This conclusion is surprising, for the coefficient of determination reported for each of the models was .999833, a value that would have lead most researchers to accept either of the models as providing full explanation for the quarterly variability of consumption over almost half a century.

The third example is found in Davidson and MacKinnon [1981] where they report on the results of applying the J-test to the five alternative consumption function models examined by Pesaran and Deaton [1978]. In spite of the fact that the coefficients of determination for all the models are quite high, ranging from .997933 to .998756 [Pesaran and Deaton, 1978, 689-91], each of the models is rejected against one or more of the alternatives.

In this paper we show that when we wish to test alternative non-nested specifications that are successful in explaining the observed variations, the J-test is likely to be inconclusive. While advances and improvements on the J-test such as the Fast Double Bootstrap procedure

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<sup>4</sup> That is to say that each of the alternative hypotheses fit the data extremely well, where the regressors of the alternative hypotheses are correlated, where the alternatives have the same numbers of regressors, J-test is inconclusive.

[Davidson and MacKinnon, 2002] have been made and are reported to increase the power of the test, it appears that in doing empirical work researchers still use the standard J-test[see for examples: Faff and Gray (2006) or Singh (2004)].

In the discussion below we use the original test as this allows for clarity of exposition.

In section 2, we point out the theoretical reasons why the test may lack power in testing model specifications that fit a given set of data well. We do this by expressing the test statistic in terms of the correlation between the variables in the alternative specification. In section 3, we illustrate the problems encountered in using the J-test in empirical work by applying the test to two alternative specifications designed to explain monthly output behavior in 24 industries.

In section 4, we present a testing paradigm (Surekha, Ghali and Krieg, 2008) for non-nested hypothesis that can be implemented to supplement the J-test when the J-test proves inconclusive. We use log-likelihood values which are obtained in the process of applying the J-test to approximate Bayesian information criteria and Bayes factors. This allows us to circumvent the complexities of the Bayesian approach, namely that of specifying the prior distributions and computations of marginal likelihoods. This specification testing method yields results that do not depend on the choice of the null or the maintained hypothesis. We provide empirical application of the Bayesian extension of the J-test by applying it to the same data on the 24 industries studied in section III, to enable us to contrast the results of our procedure with the J-test. Finally we discuss some public policy specifications that may be studied using the proposed Bayesian extension to the J-test.

## 2. The J-Test

An “artificial regression” approach for testing non-nested models was proposed by Davidson and MacKinnon [1981, 1993]. Consider two non-nested hypotheses that are offered as alternative explanations for  $Y$ :

$$H_0: Y = X\beta + \varepsilon_1, \text{ and} \quad \dots (2.1)$$

$$H_1: Y = Z\gamma + \varepsilon_2, \quad \dots (2.2)$$

The error terms  $\varepsilon_1$ ,  $\varepsilon_2$  satisfy the classical normal model assumptions,  $X$  has  $k_1$  and  $Z$  has  $k_2$  independent non-stochastic regressors.

We write the artificial compound model as:

$$Y = (1 - \alpha)X\beta + \alpha Z\gamma + \varepsilon \quad \dots(2.3)$$

If this model in (2.3) is estimated, we test the non-nested model by testing one parameter: when  $\alpha=0$ , the compound model collapses to equation (2.1) and when  $\alpha = 1$ , the compound model collapses to equation (2.2).

Because the parameters of this model are not identifiable, Davidson and MacKinnon suggest replacing the compound model (2.3) by one “in which the unknown parameters of the model *not* being tested are replaced by estimates of those parameters that would be consistent if the DGP [data generating process] actually belonged to the model they are defined” (Davidson and MacKinnon, 1993, p. 382). Thus, to test equation (2.1), we replace  $\gamma$  in (2.3) by its estimate  $\hat{\gamma}$  obtained by regressing  $Y$  on  $Z$ . Further, if we rewrite  $\hat{Y}_z = Z\hat{\gamma}$ , the equation to be estimated to test whether  $\alpha=0$  is:

$$Y = (1 - \alpha)X_1\beta + \alpha\hat{Y}_z + \varepsilon \quad \dots (2.3')$$

Similarly, to test equation (2.2) we estimate  $\hat{\beta}$  by fitting equation (2.1) to the data and replace  $X\beta$  in (2.3) by  $X\hat{\beta} = \hat{Y}_x$ . The equation to be estimated to test (2.2) is then,

$$Y = (1 - \alpha)\hat{Y}_x + \alpha ZY + \varepsilon \quad \dots (2.3'')$$

The J-test applies the standard Student's t-test for the estimated coefficients of  $\hat{Y}_z$  in equation (2.3') and  $\hat{Y}_x$  in equation (2.3''). A statistically significant t-statistic on the coefficient of  $\hat{Y}_z$  rejects  $H_0$  as the appropriate model and a significant t-statistic on the  $\hat{Y}_x$  coefficient results in the rejection of  $H_1$ . For the consumption function examples described earlier in the introduction, both t-statistics result in the rejection of each model. As some of the regressors in (2.3') and (2.3'') are stochastic, the t-test is not strictly valid, a point noted by Davidson and MacKinnon who show that the J and P tests (which in [linear models] are identical) are asymptotically valid.<sup>5</sup>

In this section we show that the t-test statistic for the significance of  $\hat{\alpha}$  in (2.3'), thus the decision we make regarding the hypothesis (2.1), depends on the goodness of fit of the regression of Y on Z, the goodness of fit of the regression (2.3') as well as the correlation between the two sets of regressors in (2.3'). We show this using the F ratio for testing  $\alpha=0$ , which is identical to the square of the t-value since we are interested in the contribution of only one regressor  $\hat{Y}_z$ . A similar statement applies to the test of the significance of  $(1 - \alpha)$  in (2.3'').

Consider the OLS estimator of the coefficient  $\alpha$  of the model (2.3'). Using a theorem due to Lovell (1963, p. 1001),<sup>6</sup> the OLS estimate of  $\alpha$  and the estimated residuals will be the same as those obtained from regressing the residuals of the regression of Y on X,  $M_x Y$ , on the residuals of regressing  $\hat{Y}_z$  on X,  $M_x \hat{Y}_z$ :

$$M_x Y = \alpha M_x \hat{Y}_z + M_x \varepsilon \quad \dots (2.4)$$

Where,  $M_x = [I - X(X'X)^{-1}X']$ , and  $\hat{Y}_z = Z\hat{\gamma}$  and  $\hat{\gamma}$  is the estimated regression coefficients of Y on Z:

Writing,  $P_z = Z(Z'Z)^{-1}Z'$ , we write (2.4) as:

$$M_x Y = \alpha M_x P_z Y + M_x \varepsilon .$$

The OLS estimator of  $\alpha$  is then:

$$\hat{\alpha} = [Y'P_z M_x P_z Y]^{-1} Y'P_z M_x Y \quad \dots (2.5)$$

The residuals of OLS estimation of (2.4) are:

$$M_x \hat{\varepsilon} = [M_x Y - \hat{\alpha} M_x P_z Y]$$

<sup>5</sup> They also add, "also indicates why they (J and P tests) may not be well behaved in finite samples. When the sample size is small or Z contains many regressors that are not in S(X)..." We do not consider these situations in what follows.

<sup>6</sup> Lovell's theorem 4.1 generalizes (to deal with seasonal adjustment) a theorem that was developed by R. Frisch and F. Waugh for dealing with detrending data. Green (2003) extends the application to any partitioned set of regressors.

Since (2.4) has only one regressor, under the null hypothesis that  $\alpha = 0$  the  $F$ -statistic is the square of the  $t$ -statistic.

The sum of squares due to regression of equation (2.4),  $Q$ , is given by:

$$Q = \hat{\alpha}' D \hat{\alpha}, \text{ where } D = \hat{Y}'_z M_x \hat{Y}_z \quad \dots (2.6)$$

Consider regressing  $Y$  on  $X$  only and denote the residuals of that regression by  $\hat{u} = M_x Y$  and regressing  $\hat{Y}_z$  on  $X$  and denote the residuals of that regression by:  $\hat{v} = M_x \hat{Y}_z$ . We can then write:

$$\hat{\alpha} = (\hat{v}' \hat{v})^{-1} \hat{v}' \hat{u}, \text{ and} \quad \dots (2.5')$$

$$Q = Y' M_x \hat{Y}_z [\hat{Y}'_z M_x \hat{Y}_z]^{-1} \hat{Y}'_z M_x Y = \hat{u}' \hat{v} (\hat{v}' \hat{v})^{-1} \hat{u}' \hat{v} = (\sum \hat{u} \hat{v})^2 / \sum \hat{v}^2 \quad \dots (2.6')$$

The residuals from OLS estimation of (2.4) can be written as:

$$M_x \hat{\varepsilon} = [M_x Y - \hat{\alpha} M_x P_z Y] = \hat{u} - \hat{\alpha} \hat{v} = \hat{u} - (\hat{v}' \hat{v})^{-1} (\hat{v}' \hat{u}) \hat{v}$$

The sum of the squared residuals from estimating (2.4) is:

$$\hat{\varepsilon}' M_x \hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u} \hat{v})^2 / \sum \hat{v}^2] \quad \dots (2.7)$$

This sum of squares has  $(T - k_1 - 1)$  degrees of freedom, where  $T$  is the number of observations and  $k_1$  is the number of variables in  $X$ .

Thus, under the hypothesis that  $\alpha = 0$ , the  $F$ -statistic is:<sup>7</sup>

$$F(1, T - k_1 - 1) = Q / [\hat{\varepsilon}' M_x \hat{\varepsilon} / (T - k_1 - 1)] = \left[ \frac{(\sum \hat{u} \hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u} \hat{v})^2} \right] (T - k_1 - 1) \quad \dots (2.9)$$

This test statistic can be expressed in terms of correlations between the variables. We show in the Appendix that:

$${}^2F(1, T - k_1 - 1) = \frac{(T - k_1 - 1) [R_{yz} - R_{yx} R_{\hat{y}_z}]^2}{(1 - R_{yx}^2)(1 - R_{\hat{y}_z}^2) - [R_{yz} - R_{yx} R_{\hat{y}_z}]^2} \quad \dots (2.10)$$

Where we placed the superscript 2 to denote that it is a test for the second model, equation (2.2), under the assumption that the first model is true, and where:

$R_{yx}^2$  is the coefficient of determination of the regression of  $Y$  on  $X$  only,

$R_{yz}^2$  is the coefficient of determination of the regression of  $Y$  on  $Z$  only,

$R_{\hat{y}_z}^2$  is the coefficient of determination of the regression of  $\hat{Y}_z$  on  $X$ ,

<sup>7</sup> See equation (22) of Godfrey and Peseran (1983).

$R_{\hat{y}_x \hat{y}_z}$  is the correlation coefficient of  $\hat{Y}_x$  and  $\hat{Y}_z$ , and since these are linear combinations of  $X$  and  $Z$  respectively,  $R_{\hat{y}_x \hat{y}_z}$  is the canonical correlation of the alternative regressors  $X$  and  $Z$ .<sup>8</sup>

Because the  $J$  test is symmetric, the second part of the  $J$  test, maintaining (2.2) and testing for the significance of  $(1 - \alpha)$  in (2.3"), the test statistic, denoted as  ${}^1F$  is:

$${}^1F(1, T - k_2 - 1) = \frac{(T - k_2 - 1)[R_{yx} - R_{yz}R_{\hat{y}_x \hat{y}_z}]^2}{(1 - R_{yz}^2)(1 - R_{\hat{y}_z}^2) - [R_{yx} - R_{yz}R_{\hat{y}_x \hat{y}_z}]^2} \quad \dots(2.11)$$

From these two test statistics we note the following:

When sample size is small, the difference between the numbers of regressors in the competing model will affect the sizes of the test statistics. If the data were generated by the model of (2.1), the test statistic  ${}^1F$  will get smaller as the number of regressors in the alternative model,  $k_2$ , increases. This may lead to the rejection of (2.1) in favor of the alternative model (2.2), particularly if the number of regressors  $k_1$  is small. This is consistent with Godfrey and Pesaran's (1982) simulation-based findings as well as with Gourieroux and Monfort (1994) who conclude that the test "is very sensitive to the relative number of regressors in the two hypotheses; in particular the power of the  $J$ -test is poor when the number of regressors in the null hypothesis is smaller than the number of regressors in the alternative one." However, the influence of the differentials in the number of regressors will become negligible as sample size increases.

When a model is successful in explaining the variations in  $Y$  the  $J$ -test is likely to reject it. To see this clearly, assume that the alternative regressors are orthogonal so that  $R_{\hat{y}_x \hat{y}_z} = 0$ . If model (2.1) is successful, the high coefficient of determination  $R_{yx}^2$  will increase the numerator of (2.11) while reducing the denominator, thus increasing the value of the test statistic  ${}^1F$  which leads to rejection of the model (2.1). Similarly, if the model (2.2) is successful in explaining the variations in  $Y$ , the high value of  $R_{yz}^2$  will increase the value of the test statistic  ${}^2F$  which leads to the rejection of model (2.2). When both models are successful in explaining the variations in  $Y$ , the combined effect of high  $R_{yx}^2$  and  $R_{yz}^2$  lead us to reject both models. Such was the situation in Davidson and MacKinnon[1981] report on the five alternative consumption functions where the coefficients of determination for all the models ranged from .997933 to .998756, yet all the models were rejected. This would be at variance with the conclusion reached by Godfrey and Pesaran (1983) "when sample sizes are small the application of the (unadjusted) Cox test or the  $J$ -test to non-nested linear regression models is most likely to result in over rejection of the null hypothesis, even when it happens to be true, if ...the true model fits poorly", unless the fit of the false model also fits poorly.

<sup>8</sup> Where there is only one regressor in each of  $X$  and  $Z$ , the coefficient  $R_{\hat{y}_x \hat{y}_z}$  is the correlation between the two regressors and the test statistic simplifies to:

$${}^2F(1, T - k_1 - 1) = \frac{(T - k_1 - 1)[R_{yz} - R_{yx}R_{xz}]^2}{1 - R_{xz}^2 - R_{yx}^2 - R_{yz}^2 + 2R_{yz}R_{yx}R_{xz}} .$$

$R_{\hat{y}_x \hat{y}_z}$  is the canonical correlation coefficient of the sets of regressors X and Z. Higher values of this correlation would reduce the numerator and increase the denominator of both (2.10) and (2.11), lowering the values of the F statistics. The effect would reduce the likelihood of rejecting either of the competing hypotheses. The reverse, as stated in Godfrey and Pesaran (1983) is also true: “when the correlation among the regressors of the two models is weak” the J-test “is most likely to result in over rejection of the null hypothesis, even when it happens to be true,”

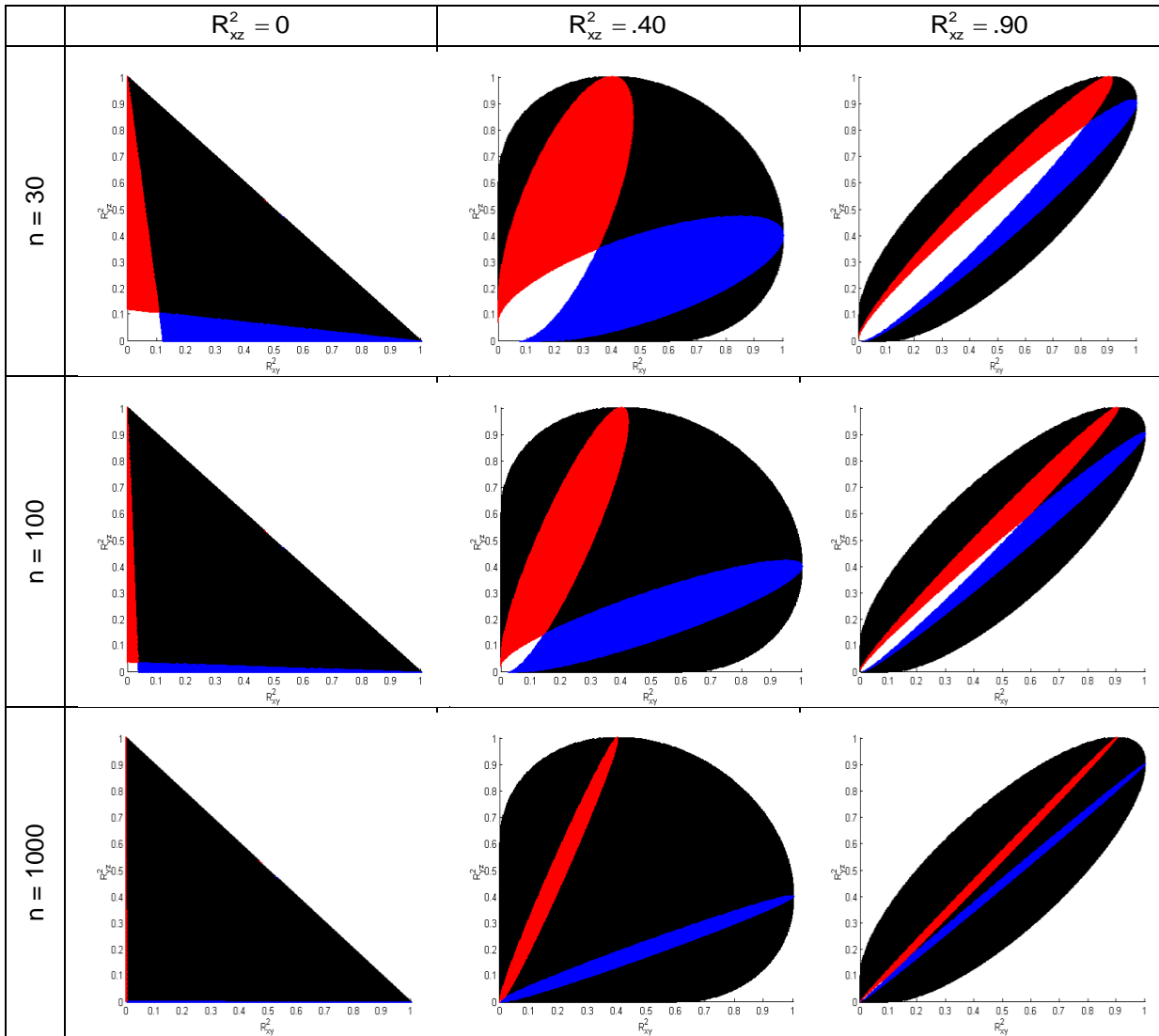
The effect of the coefficients of determination of the alternative model specifications on the F statistic is shown in Figure 1.<sup>9</sup> In this figure, the light grey areas represent combinations of  $R_{yx}^2$  and  $R_{yz}^2$  that result in rejecting the X model (2.1) and failing to reject the Z model (2.2). This is appropriate since for those combinations, the model using the set of explanatory variables Z is clearly superior to that which uses the set X. The dark grey areas represent combinations that result in rejecting the model that uses the set of explanatory variables Z and failing to reject the model which uses X. Again, this is clearly appropriate. The interior white areas represent combinations of the coefficients of determination for which the J-test fails to reject both models. Within those areas comparisons of the coefficients of determination for the two alternative models, particularly for large samples would have led to the conclusion that neither model is particularly useful in explaining Y. The black areas represent combinations of  $R_{yx}^2$  and  $R_{yz}^2$  that result in rejecting both hypotheses. What is remarkable is the size of these areas compared to the other areas and the fact that the black area encompasses combinations of  $R_{yx}^2$  and  $R_{yz}^2$  that, because of their large difference, would reasonably preclude a researcher from employing the J test.<sup>10</sup> For instance, consider two competing models in the middle panel (the case of  $n = 100$  and  $R_{yz}^2 = .4$ ). If one model had  $R_{yx}^2 = .9$  and the other  $R_{yz}^2 = .6$ , the J- test would reject both models despite the fact that the X model would be traditionally viewed as the superior model based solely on the comparisons of the coefficients of determination.

The canonical correlation of the competing model's independent variables impacts the permissible values of the J- test. The first panel demonstrates a canonical correlation between regressors set at zero, in the second panel it is set at .40 and in the third it is set at .90. It is

<sup>9</sup> The coefficients of determination and the canonical correlations are subject to restrictions. Since the quadratic form  $\hat{\varepsilon}'M_x\hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u}\hat{v})^2 / \sum \hat{v}^2]$  is positive semi-definite,  $\sum \hat{u}^2 \sum \hat{v}^2 \geq (\sum \hat{u}\hat{v})^2$ , that is:  $(1 - R_{yx}^2)(1 - R_{yz}^2) \geq [R_{yz} - R_{yx}R_{\hat{y}_x \hat{y}_z}]^2$ . The restrictions imply that when the canonical correlation between X and Z is zero (the two sets of alternative explanatory variables are orthogonal),  $R_{yx}^2 + R_{yz}^2 \leq 1$ . Thus, in figures (1.a), (1.b) and (1.c) where the canonical correlation is set at zero, the only feasible region is the triangle below the line connecting the points  $R_{yx}^2 = 1$  and  $R_{yz}^2 = 1$ . When the canonical correlation is different from zero, the restriction on the relationship between the coefficients of determination result in the elliptical shape of the feasible region shown in the second and third columns of Figure 1. Combinations of the coefficients of determinations outside of the ellipse violate the restriction.

<sup>10</sup> The white spaces outside of the shaded areas are regions where the combinations of the coefficients of determination that are not permissible- they result in violating the requirement that  $\hat{\varepsilon}'M_x\hat{\varepsilon} = \sum \hat{u}^2 - [(\sum \hat{u}\hat{v})^2 / \sum \hat{v}^2]$  is positive semi-definite.

worth noting that when the canonical correlation is greater than zero, the size of the permissible region decreases as the correlation increases. In the extreme case where the canonical correlation approaches 1, so that each of the variables X and Z are a linear combination of the other, the permissible combinations of the coefficients of determination,  $R^2_{xy}$  and  $R^2_{yz}$  will lie on the 45 degree diagonal emanating from the origin.



Notes: Black area represents reject both region, red (light grey) area represents reject the X model and fail to reject the Z model, blue (dark grey) represents reject the Z and fail to reject the X model, interior white areas represents fail to reject both models. All graphs were produced at a 5% level of significance.

**Figure 1. J-Test Results for Various N,  $R^2_{xz}$ ,  $R^2_{xy}$ , and  $R^2_{yz}$**

The effect of sample size on both the permissible region, we present the figures for sample sizes 30, 100 and 1000 in each of the three panels. The size of the permissible region



depends only on the value of the canonical correlation, and is independent of sample size as would be expected. It is clear from these figures that as sample size increases, the area in which the J-test would lead to the rejection of both hypotheses expands and thus covers increasingly larger areas of the permissible region. In the next section we present empirical example based on data for the select US industries to examine the issues with J-test in empirical applications. Our analysis shows yet another example of non-nested econometric models where the J-test fails to choose either model uniquely.

### 3. Determinants of Monthly Variations in Industry Output

#### 3.1 Alternative Model Specifications for Production Behavior

We now apply the J-test to compare two model specifications that have been used to explain monthly variations in industry output (Ghali, 2005). In both specifications monthly output is determined by sales. In one specification the stock of inventories also influences production, while in the other specification inventory stock does not play a role. The two specifications also differ in the way in which the variable sales enter into the specification.

Minimizing the discounted cost over an infinite horizon for the traditional cost function used by many researchers results in the Euler equation reported by Ramey and West (1999, p. 885). Solving for current period output,  $Q_t$  and assuming the cost shocks to be random,<sup>11</sup> we get the following equations:

$$Q_t = \beta_0 + \beta_1[\Delta Q_t - 2b\Delta Q_{t-1} + b^2\Delta Q_{t-2}] + \beta_2 Q_{t-1} + \beta_3 S_{t-1} + \beta_4 H_t + u_t \quad \dots (3.1)$$

where  $Q_t$  is output in month “ $t$ ”,  $S_t$  is sales and  $H_t$  is the inventory stock at the end of the month.

The minimization of the cost using an alternative cost function (Ghali, 1987) and solving the resulting Euler equation for output we get:

$$Q_{it} = \gamma_0 + \gamma_1 S_{it} + \gamma_2 \bar{S}_t + u_{it} \quad \dots (3.2)$$

where  $S_{it}$  represents sales in month “ $t$ ” of production planning horizon “ $t$ ” and  $\bar{S}_t$  is the average sales over the production planning horizon.

We apply the J-test to choose between the two non-nested model specifications, M1 in equation (3.1) and M2 in equations (3.2) as each explains the monthly variability of production.

#### 3.2 The Data

The data we use are those used by Krane and Braun (1991).<sup>12</sup> These data are in physical quantities, thus obviating the need to convert value data to quantity data and eliminating the

<sup>11</sup> The empirical justification for this assumption is that the estimates reported in the literature for the effect of factor price variations on cost are not strongly supportive of the assumption that the cost shocks are observable. Ramey and West (1999) tabulated the results of seven studies regarding the significance of the estimated coefficients for input prices. They reported that wages had a significant coefficient in only one study and material prices in one study. (Ramey and West, 1999, 907). More detailed discussion is given in Ghali (2004).

<sup>12</sup> We are very grateful to Spencer D. Krane who made this data available.

numerous sources of error involved in such calculations.<sup>13</sup> The data are monthly, eliminating the potential biases that may result from temporal aggregation.<sup>14</sup> They are at the four-digit SIC level or higher, reducing the potential biases that may result from the aggregation of heterogeneous industries into the two-digit SIC level.<sup>15</sup> The data are not seasonally adjusted, thus obviating the need for re-introducing seasonals in an adjusted series.<sup>16</sup> A description of the data and their sources is available in Table 1 of Krane and Braun (1991, 564-565). We use the data on the 24 industries studied by Ghali (2005).

*Empirical Results of the J-test*<sup>17</sup>

In Table 1 the results of applying the *J* test to compare the two specifications are reported. In the first set of columns we report the results of testing M1, equation (3.1) assuming that M2, equation (3.2) is maintained. This is done by estimating the parameters of equation (3.2) using OLS as suggested by Davidson and MacKinnon,<sup>18</sup> The coefficient of determination,  $R^2$ , of those regressions are reported in the first column. We then used the predicted values of  $Q_i$  from that regression as an added regressor in the estimation of equation (1'). The coefficient of the added regressor,  $\hat{Q}_i$ , is reported in the second column and its *t* value in the third column. If the coefficient of the added regressor is significantly different from zero, the model specification (3.1) is rejected in favor of the model specification (3.2). As the fourth column shows, this was the case for all of the industries.

The process is reversed in the second set of columns of Table 1. We now maintain the model specification of equation (3.1) and test that of equation (3.2). The last column of this set of columns shows that the model of equation (3.2) is rejected in favor of the model specification of equation (3.1).

As can be seen from Table 1, for all industries studied both competing specifications are rejected by the *J*-test. "When both models are rejected, we must conclude that neither model is satisfactory, a result that may not be welcome but that will perhaps spur us to develop better models."(Davidson and MacKinnon, 1993, p. 383). However, it should be noted that each of the model specifications explains very high proportion of the monthly variation of output that as seen by the high coefficients of determination reported for each. It may be that because each of the specifications is so successful in explaining the behavior of output, the *J*-test is not able to distinguish between them. In other words, if the maintained specification is successful in explaining the dependant variable, the correlation between the predicted value and the dependant value will be significant, and so will be the coefficient of the predicted value when added as a regressor in the artificial compound model. This weakness of the *J*-test has been noted in the literature and we explain in detail the theoretical reasons for this kind of problem.

<sup>13</sup> For discussion of the potential measurement errors in converting value to quantity data for inventory stocks, see Krane and Braun (1991, 560–562).

<sup>14</sup> See Ghali (1987) and Lai (1991).

<sup>15</sup> For discussion of the potential biases see Krane and Braun (1990, 7).

<sup>16</sup> For example see Ramey (1991). She had to re-introduce seasonality as the data she was using was seasonally adjusted.

<sup>17</sup> The results reported in Table (1) are from Ghali 2007 reproduced with permission from publisher.

<sup>18</sup> All equations were estimated under the assumption of an AR(1) process for the error term.

**TABLE 1. Results from Davidson- MacKinnonJ -TEST**

INDUSTRY	SAMPLE PERIOD	H <sub>2</sub> Maintained, H <sub>1</sub> Tested				H <sub>1</sub> Maintained H <sub>2</sub> Tested			
		R <sub>2</sub> <sup>2</sup>	$\alpha$	<i>t</i>	H <sub>1</sub>	R <sub>1</sub> <sup>2</sup>	$\alpha$	<i>t</i>	H <sub>2</sub>
Asphalt1	1977:01-1988:12	.955	.767	16.76	R	.980	.959	14.98	R
Bituminous Coal	1977:01-1988:09	.606	.371	8.432	R.	.944	.958	34.276	R
Cotton Fabric	1975:01-1986:12	.936	.507	13.201	R.	.944	.578	14.242	R
Distillate Fuel	1977:01-1988:12	.686	.330	7.000	R	.943	1.060	28.310	R
Gasoline	1977:01-1988:12	.733	.246	3.750	R	.904	1.058	19.085	R
Glass Containers	1977:01-1989:03	.670	.231	4.301	R	.902	.969	20.822	R
Iron and Steel Scrap	1956:01-1988:12	.969	.596	15.868	R	.966	.642	18.185	R
IronOre	1961:01-1988:12	.873	.376	12.489	R	.966	1.087	37.225	R
Jet Fuel	1977:01-1988:12	.899	.182	3.932	R	.983	.961	28.234	R
Kerosene	1977:01-1989:03	.823	.318	5.551	R	.949	1.155	26.824	R
Liquefied Gas	1977:01-1988:12	.376	.441	7.015	R	.917	1.045	31.742	R
Lubricants	1977:01-1988:12	.496	.198	2.940	R	.912	1.046	25.697	R
Man-made Fabric	1975:01-1986:12	.904	.587	11.894	R	.903	.553	10.606	R
Newsprint Canada	1961:01-1988:12	.838	.243	8.769	R	.966	.965	44.514	R
Newsprint US	1961:01-1988:12	.991	.402	12.963	R	.996	.773	30.423	R
Petroleum Coke	1977:01-1989:03	.933	.216	5.377	R	.983	.847	20.469	R
Pig Iron	1961:01-1988:12	.997	.924	39.214	R	.985	.242	8.294	R
Pneumatic Casings	1966:01-1988:12	.802	.317	12.310	R	.956	.983	40.722	R
Residual Fuel	1977:01-1988:12	.965	.313	6.950	R	.991	.997	29.659	R
Slab Zinc	1977:01-1988:03	.888	.254	5.283	R	.976	1.065	28.974	R
Sulfur	1961:01-1988:12	.889	.281	6.348	R	.988	1.024	72.875	R
Super Phosphates	1981:01-1988:12	.941	.325	6.743	R	.981	.936	21.897	R
Synthetic Rubber	1961:01-1984:12	.833	.198	5.567	R	.965	1.026	34.948	R
Waste Paper	1977:01-1988:02	.874	.440	6.253	R	.962	.896	19.073	R

R→ Reject the Null Hypothesis

Once we realized the problem, we then explored the Bayesian test paradigm that has a unified approach for all kinds of testing situations. The Bayesian posterior odds and Bayes Factors have been used for all forms of tests and model selection problems. While the Bayesian approach is highly attractive and can be applied for nested and non-nested models, the approach is not popular in applied economics due to the difficulty in specifying the prior distributions and the complex derivation of the marginal posterior distributions. Therefore, following Surekha, Ghali and Krieg (2008) we use the Bayesian extension of the J-test to approximate Bayes' factor that provides a more practical test for non-nested hypothesis, has a Bayesian justification and yields more conclusive results for the production model. We also note that several new approaches have been developed for comparing non-nested theories in public policy and social sciences (see, Bozdogan (1987), Vuong(1989), Kiefer and Choi, (2008), Clarke (2001, 2007), and Chen(2008) ). What we propose is one of the most practical ways to extend the J-test.

#### 4. A Bayesian Extension

In many non-standard testing of hypotheses situations when the classical procedures lead to inconsistent results as in the case of J-test, the Bayesian approach provides an alternative that is consistent (see, for example; Zellner (1971, 1985, 1994), Berger and Pericchi (2001)). The Bayesian paradigm is generally more involved as it necessitates the specification of prior distribution for the parameters as well as the hypotheses, obtaining marginal likelihoods, Bayesian posterior odds and Bayes factors for the competing hypotheses. Therefore, it is not surprising that we find a rather limited number of applications of the Bayesian approach even though it is intuitively more appealing and provides consistent and meaningful results.

##### 4.1 Bayesian Hypothesis Testing for Nested and Non-nested hypotheses

The theory of Bayesian testing of hypotheses is built around the concept of posterior probabilities of hypotheses and the Bayes factor, which were first introduced by Jeffreys (1935, 1961). Bayesian model comparison concepts and the issues that arise in empirical applications have been discussed by Zellner (1971), Kass and Raftery (1995), Berger and Pericchi (2001) and Koop (2003), amongst many others. Schwarz (1978) paved the way for interplay between the Information Criteria and the Bayes factor for Bayesian specification test. We use Schwarz' approximation of Bayesian information criteria and the log likelihood values to calculate Bayes factors for the competing models.

If  $M_1, M_2$  are two different model specifications<sup>19</sup> for a given data  $D$ , the posterior odds ratio  $K_{12}$  is given by

$$K_{12} = [P(D/H_1)/P(D/H_2)] * [P(H_1) / P(H_2)] \quad \dots (4.1)$$

Or:

$$\text{Posterior Odds} = \text{Bayes factor} \times \text{Prior odds} \quad \dots (4.2)$$

In the absence of any definitive information or if we have little information we treat the two hypotheses a priori equally likely implying  $P(H_1) = P(H_2) = \frac{1}{2}$ , and the prior odds ratio

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<sup>19</sup> The two model specifications must be exhaustive if we need to obtain Posterior probabilities of hypothesis from the posterior odds. The results can be easily extended for  $k$  model specifications.

$[P(H1) / P(H2)]$  is equal to 1. If prior odds equal one, from (4.2), the Posterior odds ratio is same as the Bayes factor.

$P(D/H_i, i=1,2,...k)$ , the marginal likelihood and is also known as the weighted likelihood or the predictive likelihood and is given by

$$P(D/H_i) = \int_{\theta}^{\infty} P(D/\theta_i, H_i) \pi(\theta_i/H_i) d\theta_i \quad i=1,2,...,k \quad \dots (4.4)$$

Where  $\theta_i$  is the parameter under  $H_i$  and  $\pi(\theta_i/H_i) d\theta_i$  is its prior probability density and  $\int_{\theta}^{\infty} P(D/\theta_i, H_i)$  is the probability density of  $D$  given the value of  $\theta_i$  under the hypothesis  $H_i$  or the likelihood function of  $\theta$ . In the traditional Bayesian approach, we must specify the prior distribution  $\pi(\theta_i/H_i)$  for the parameter(s)  $\theta_i$ .

The quantity  $P(D/H_i)$ , is the predictive probability of the data; that is the probability of seeing this data which is calculated before the data is observed. Bayes factor which is the ratio of these marginal probabilities of the data shows the evidence in favor of or against the hypothesis. In case of two hypotheses,  $i=1,2$ :

$$K_{12} = P(D/H_1) / P(D/H_2) \quad \dots (4.5)$$

$$K_{12} = \int_{\theta}^{\infty} P(D/\theta_1, H_1) \pi(\theta_1/H_1) d\theta_1 / \int_{\theta}^{\infty} P(D/\theta_2, H_2) \pi(\theta_2/H_2) d\theta_2 \quad \dots (4.6)$$

If  $K_{12}$  is greater than 1, the data favors Hypothesis 1 (Model M1) over Hypothesis 2 (Model M2) and if  $K_{12}$  is less than 1, the data favors Hypothesis 2 (model M2).

### 4.2 Bayes Factor, BIC and the Likelihood Values

Although Bayes factors are fairly versatile and universally applicable for specification testing, calculation of marginal likelihoods is extremely demanding and sometimes these may not even exist (Leamer, 1978). As shown in Surekha, Ghali and Krieg (2008), we use the Bayesian extension of the J-test by using the log likelihood values generated by using the J-test for the approximation of the Bayes factor. This proves very effective in obtaining unequivocal choice of a production model. This is how it works in practice:

$$-2 \text{ SIC} \cong \text{BIC} \quad \dots (4.7)$$

$$\text{SIC} = (\log[p(D/\hat{\theta}_1, M1)] - \log[p(D/\hat{\theta}_2, M2)]) - \frac{1}{2}(p_1 - p_2) \log(n), \quad \dots (4.8)$$

Where  $\hat{\theta}_i, i=1,2$  are the MLE under Model M1 and M2,  $p_1$ , and  $p_2$  are the number of parameters in models 1 and 2 respectively and  $n$  is the sample size.

$$\text{BIC}(M1) = 2 \log p(D/\hat{\theta}_1, M1) - p_1 \ln(n) \quad \dots (4.9)$$

$$\text{BIC}(M2) = 2 \log p[D/\hat{\theta}_2, M2] - p_2 \ln(n) \quad \text{and} \quad \dots (4.10)$$

$$2 \log K_{12} = \text{BIC}(M2) - \text{BIC}(M1) \quad \dots (4.11)$$

Since BICs can be calculated from likelihood values, we can calculate twice the Bayes factor from (4.11) without specifying the prior distribution. Once we know the  $2 \log K_{12}$  and since Models M1 and M2 are exhaustive in this case we can obtain posterior probabilities  $\Pi_1$  and  $\Pi_2$  for Models M1 and M2 by using the relationship:

$$\Pi_1 = \frac{K_{12}}{1+K_{12}} ; \quad \text{and} \quad \Pi_2 = \frac{1}{1+K_{12}} \quad \dots (4.12)$$

**Table 2. Bayes factors and Posterior Probabilities of Models M1 and M2**

	<i>2 Log K21</i>	<i>K21</i>	<i>Evidence against M1</i>	<i>2 log (K12)</i>	<i>K12</i>	<i>Evidence Against M2</i>	<i>I11</i>	<i>I12</i>
Asphalt	101.99	1.4E+22	very Strong	-101.99	7.12E-23	Not worth mention	7.12E-23	1
Beer	18.56	10695.36	very Strong	-18.56	9.35E-05	Not worth mention	9.35E-05	0.999907
Bituminous Coal	322.56	1.1E+70	very Strong	-322.56	9.06E-71	Not worth mention	9.06E-71	1
Cotton Fabric	25.09	280624.6	very Strong	-25.09	3.56E-06	Not worth mention	3.56E-06	0.999996
Distillate Fuel	249.23	1.32E+54	very Strong	-249.23	7.6E-55	Not worth mention	7.6E-55	1
Gasoline	154.64	3.79E+33	very Strong	-154.64	2.64E-34	Not worth mention	2.64E-34	1
Glass Containers	178.87	6.94E+38	very Strong	-178.87	1.44E-39	Not worth mention	1.44E-39	1
Iron Scrap	-18.37	0.000103	<b>not worth mention</b>	18.37	9739.505	Very strong	0.999897	<b>0.000103</b>
IronOre	353.21	4.99E+76	very Strong	-353.21	2E-77	Not worth mention	2E-77	1
Jet Fuel	257.77	9.43E+55	very Strong	-257.77	1.06E-56	Not worth mention	1.06E-56	1
Kerosene	175.58	1.34E+38	very Strong	-175.58	7.47E-39	Not worth mention	7.47E-39	1
Liquified Gas	277.70	2E+60	very Strong	-277.70	5E-61	Not worth mention	5E-61	1
Lubricants	232.60	3.22E+50	very Strong	-232.60	3.11E-51	Not worth mention	3.11E-51	1
Man-made Fabric	7.61	44.98217	<b>Strong</b>	-7.61	0.022231	Not worth mention	0.021748	<b>0.978252</b>
Newsprint Canada	520.39	1E+113	very Strong	-520.39	1E-113	Not worth mention	1E-113	1
Newsprint US ARD	513.09	2.6E+111	very Strong	-513.09	3.8E-112	Not worth mention	3.8E-112	1
Newsprint US	276.53	1.12E+60	very Strong	-276.53	8.97E-61	Not worth mention	8.97E-61	1
Petroleum Coke	125.52	1.81E+27	very Strong	-125.52	5.53E-28	Not worth mention	5.53E-28	1
Pig Iron	-508.78	3.3E-111	<b>not worth mentioning</b>	508.78	3E+110	Very strong	1	<b>3.3E-111</b>
Pneumatic casings	512.75	2.2E+111	very Strong	-512.75	4.5E-112	Not worth mention	4.5E-112	1
Residual Fuel	197.12	6.36E+42	very Strong	-197.12	1.57E-43	Not worth mention	1.57E-43	1
Slab Zinc	216.21	8.88E+46	very Strong	-216.21	1.13E-47	Not worth mention	1.13E-47	1
Sulphur	722.68	8.5E+156	very Strong	-722.68	1.2E-157	Not worth mention	1.2E-157	1
Super Phosphate	148.91	2.17E+32	very Strong	-148.91	4.62E-33	Not worth mention	4.62E-33	1
Synthetic Rubber	460.03	7.8E+99	very Strong	-460.03	1.3E-100	Not worth mention	1.3E-100	1
Waste Paper	180.21	1.36E+39	very Strong	-180.21	7.37E-40	Not worth mention	7.37E-40	1

We use Jeffreys' (1961, appendix B) rules of thumb for interpreting Bayes factor and make decisions for choosing between two cost functions for all twenty five industries in our data set.

### 4.3 Bayes Factors and the Non-nested Models

Let us consider that the model specification  $M_1$  in equation 3.1 is the Null Hypothesis  $H_0$  and the maintained hypothesis is model specification  $M_2$  in equation 3.2. As described above, the Bayes factor  $K_{12}$  will measure the evidence for Model 1 against model 2 and  $K_{21}$  will measure the evidence against  $M_1$ . We have computed both sets of results for comparison with the performance of the J-test in the earlier section. These results are given in the Table 2 above. The results are quite consistent and unequivocal that specification  $M_2$  in equation 3.2 is strongly supported by the data for 23 of the 25 industries (except, iron scrap and pig iron) irrespective of the choice of the Null and the maintained hypotheses. These results are quite distinctive as compared to the results of the J-test and strongly support the Bayesian extension of the J-test for obtaining meaningful model comparison results for applied researchers.

## IV. Conclusion

In earlier research the original J-test has been shown to over reject when the true model fits the data poorly, when the regressors in the models being compared are highly correlated, or when the false model contains more regressors than the true model. We presented examples where the alternative specifications fit the data well but the J- test did not distinguish between them: the J- test either rejects, or fails to reject both specifications. When such situations arise, we can supplement the J-test by a Bayesian extension that uses the estimated maximum likelihood values obtained in the process of conducting the J-test. A comparison of results in Table I and Table II clearly demonstrate that a Bayesian extension of the J-test provides a nice and practical test for non-nested hypothesis that yields unequivocal results. This Bayesian extension of the J-test that we demonstrate in case of production models has an advantage as it gives us all the benefits of the Bayesian paradigm and the Bayes factors without having to specify prior probabilities and going through the extensive Bayesian computations. Further work is needed to show the robustness of the Bayesian extension of the J-test for non-nested models and a comparison with other information based methods for non-nested specifications.

### APPENDIX

Expressing the  $F$  statistic in terms of correlations between the variables

$$F(1, T-k_1-1) = Q / [\hat{\varepsilon}' M_x \hat{\varepsilon} / (T-k_1-1)] = \left[ \frac{(\sum \hat{u}\hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u}\hat{v})^2} \right] (T-k_1-1)$$

This test statistic can be expressed in terms of correlations between the variables. For simplicity, we assume that all variables are deviations from means. Now:

$$\begin{aligned} \sum \hat{u}^2 &= \sum Y^2 - \sum \hat{Y}_x^2 = \sum Y^2 [1 - R_{yx}^2] = (T-1) s_y^2 [1 - R_{yx}^2] \\ \sum \hat{v}^2 &= \sum \hat{Y}_z^2 - \sum \hat{Y}_{zx}^2 = \sum \hat{Y}_z^2 [1 - R_{zx}^2] = (T-1) s_z^2 [1 - R_{zx}^2] \\ \hat{u}\hat{v} &= Y' M_x P_z Y = Y' P_z Y - Y' P_x P_z Y = \sum \hat{Y}_z^2 - \sum \hat{Y}_x \hat{Y}_z \\ (\hat{u}\hat{v})^2 &= [\sum \hat{Y}_z^2 - \sum \hat{Y}_x \hat{Y}_z]^2 = (\sum \hat{Y}_z^2)^2 + (\sum \hat{Y}_x \hat{Y}_z)^2 - 2 \sum \hat{Y}_z^2 \sum \hat{Y}_x \hat{Y}_z \\ &= (\sum Y^2)^2 [(\sum \hat{Y}_z^2)^2 / (\sum Y^2)^2 + (\sum \hat{Y}_x \hat{Y}_z)^2 / (\sum Y^2)^2 - 2 \{(\sum \hat{Y}_z^2) / (\sum Y^2)\} \{(\sum \hat{Y}_x \hat{Y}_z) / (\sum Y^2)\}] \\ &= (T-1)^2 s_y^4 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(\sum \hat{Y}_x \hat{Y}_z) / (\sum Y^2)] \\ &= (T-1)^2 s_y^4 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(\text{Cov}(\hat{Y}_x \hat{Y}_z) / s_y^2)] \\ &= (T-1)^2 s_y^4 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)] \end{aligned}$$

$$\begin{aligned} F &= \frac{(\sum \hat{u}\hat{v})^2}{\sum \hat{u}^2 \sum \hat{v}^2 - (\sum \hat{u}\hat{v})^2} (T-k_1-1) \\ &= \frac{(T-k_1-1)(T-1)^2 s_y^4 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]}{(T-1)^2 s_y^2 s_z^2 (1-R_{yx}^2)(1-R_{zx}^2) - [(T-1)^2 s_y^4 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]} \\ &= \frac{(T-k_1-1) s_y^2 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]}{s_y^2 (1-R_{yx}^2)(1-R_{zx}^2) - [s_y^2 R_{yz}^2 [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]} \end{aligned}$$

But  $s_{yz}^2 = s_y^2 R_{yz}^2$ , so that the expression can be written as:

$$F = \frac{(T-k_1-1)[R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]}{(1-R_{yx}^2)(1-R_{zx}^2) - [R_{yz}^2 + R_{yx}^2 \cdot R_{yz}^2 - 2(R_{\hat{y}_x \hat{y}_z} s_{yx} s_{yz} / s_y^2)]}$$

The  $F$  statistic for the model (2.2) with model (2.1) as maintained hypothesis, which we denote by  ${}^2F$  is given by:

$${}^2F(1, T-k_1-1) = \frac{(T-k_1-1)[R_{yz} - R_{yx} \cdot R_{\hat{y}_x \hat{y}_z}]^2}{(1-R_{yx}^2)(1-R_{zx}^2) - [R_{yz} - R_{yx} \cdot R_{\hat{y}_x \hat{y}_z}]^2}$$

The second part of the  $J$  test consists of maintaining (2.2) and testing for the significance of  $(1-\alpha)$  in (2.3<sup>o</sup>). This can be similarly derived with the roles of  $X$  and  $Z$  reversed. If the number of regressors in  $Z$  is  $k_2$ , the test statistic which we denote by  ${}^aF$  is:



$${}^1F(1, T - k_2 - 1) = \frac{(T - k_2 - 1)[R_{y_x} - R_{y_z} \cdot R_{\hat{y}_x \hat{y}_z}]^2}{(1 - R_{yz}^2)(1 - R_{\hat{y}_z}^2) - [R_{y_x} - R_{y_z} \cdot R_{\hat{y}_x \hat{y}_z}]^2}$$

$R_{\hat{y}_x}^2$  is the coefficient of determination of the regression of  $\hat{Y}_z$  on  $X$ . Since  $\hat{Y}_z$  is a linear transformation of  $Z$ ,  $\hat{Y}_z = Z\hat{Y}$ , the coefficient  $R_{\hat{y}_x}^2 = R_{xz}^2$ .

$R_{\hat{y}_x \hat{y}_z}$  is the correlation coefficient of  $\hat{Y}_x$  and  $\hat{Y}_z$ , and since these are linear transformations of  $X$  and  $Z$  respectively,  $R_{\hat{y}_x \hat{y}_z}$  is the canonical correlation of the alternative regressors  $X$  and  $Z$ .

If  $Z$  has only one variable and  $X$  has only one variable,  $R_{\hat{y}_x \hat{y}_z} = R_{xz}$ , and the  $F$  statistic for the model (2.2) with model (2.1) as maintained hypothesis, which we denote by  ${}^2F$  is given by:

$${}^2F(1, T - 2) = \frac{(T - 2)[R_{\hat{y}_z} - R_{xz}^2]^2}{(1 - R_{yx}^2)(1 - R_{xz}^2) - [R_{\hat{y}_z} - R_{xz}^2]^2}$$

This can be written as:

$${}^2F(1, T - 2) = \frac{(T - 2)[R_{y_z} - R_{y_x} \cdot R_{xz}]^2}{1 - R_{xz}^2 - R_{yx}^2 - R_{yz}^2 + 2R_{yz} R_{y_x} \cdot R_{xz}}$$

The second part of the J test consists of maintaining (2.2) and testing for the significance of  $(1 - \alpha)$  in (2.3"). This can be similarly derived with the roles of  $X$  and  $Z$  reversed. If the number of regressors in  $Z$  is  $k_2$ , the test statistic which we denote by  ${}^1F$  is:

$${}^1F(1, T - k_1 - 1) = \frac{(T - k_1 - 1)[R_{\hat{y}_x} - R_{xz}^2]^2}{(1 - R_{yz}^2)(1 - R_{xz}^2) - [R_{\hat{y}_x} - R_{xz}^2]^2}$$

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